CALIBRATION

Calibration of weighing instruments and uncertainty of calibration

TEUVO LAMMI, The Finnish Association of Technology for Weighing, Helsinki, Finland

Summary

In this paper the calibration of weighing instruments and the uncertainty associated with the calibration results are investigated. The main questions dealt with are:

- In order to judge the calibration results, assume that the user of an instrument has given maximum tolerable errors (MTE's) for the instrument with different loads. With their aid:
  - the person performing the calibration can decide how accurately to read the indications;
  - he/she can choose weights of sufficient accuracy for the calibration; and
  - if the errors of the instrument observed in the calibration are within the MTE's, the probability that the "true" values of the errors are within the MTEs can be given.
- The relationship between the uncertainty of calibration and its components are investigated.
- Besides the uncertainty of calibration, the uncertainty of practical weighing is outlined.

1 Introduction

This paper deals with the calibration of weighing instruments and the uncertainty of calibration and an attempt is made to arrive at an uncertainty which can be associated with the results of practical weighing. This is the uncertainty of weighing.

The weighing instruments considered here are non-automatic, single-interval instruments.

The test procedures used in the calibration and in the evaluation of the uncertainty are based on those given in OIML Recommendation R 76-1 *Nonautomatic weighing instruments. Part 1: Metrological and technical requirements - Tests*, 1992. (The references here are to OIML R 76-1 but they could as well be to the European Standard: EN 45501, AC:1993).

The evaluation of the uncertainty is based on the *Guide to the expression of uncertainty in measurement (GUM)*, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, Corrected & reprinted edition, 1995.

In Section 2 the calibration and the measures to be taken in connection with the calibration are dealt with. In Section 3 the uncertainty, its nature, practical meaning and suitable values are considered. In Section 4 the formula of the uncertainty of calibration and its components are investigated. In Section 5 examples of the application of the formula are given. In Section 6 the uncertainty of weighing is outlined.

2 Calibration of weighing instruments

2.1 What does calibration signify?

According to the definition of calibration in the *International Vocabulary of Basic and General Terms in Metrology (VIM)*, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1993, the calibration of a weighing instrument can be interpreted as being the determination of the errors in the indication of the instrument at different test loads (standard weights). The error is the difference between the indication and the value of the corresponding test load (the "true" value of the indication).

2.2 Weighing test

For the purpose of this paper a transcription of the "Weighing test" in OIML R 76-1, A.4.4.1 is presented.

2.2.1 Checks

Before the weighing test, check the leveling and ensure that electrically powered instruments have been switched on for a period of at least 0.5 h and have reached temperature stability. Preload the instrument to a "large" load and check the function of the instrument. Unload and allow the instrument to recover before the weighing test.

2.2.2 Test

Set the indication to zero at no-load. Apply test loads (standard weights, see 2.3.4) from zero up to Max of the instrument (Max = the maximum weighing capacity) and similarly remove the test loads back to zero. At least five test loads are used. The indication is read at each load.

The first test load should be applied centrally to the load receptor and the subsequent loads should be uniformly distributed on the load receptor around its midpoint.

This is the calibration test (the comparison of the measured values with the "true" values). The results can be presented by a graph with the load on the abscissa axis and the error of the instrument (2.1) on the ordinate axis. This graph is the "weighing curve".

2.3 Measures to be taken in connection with calibration

2.3.1 Maximum tolerable errors (MTEs) and f

Suppose that the user of the instrument has selected an error f. With its aid he can define the "maximum tolerable errors", MTEs, of the instrument (compare f with e in OIML R 76-1, 3.5). For example, the MTEs can be:

- ±1 f for all the loads, or
- ±0.5 f for certain "small" loads but ±1 f for larger loads, or
- ±0.5 f for "small" loads, ±1 f for certain "medium" loads and ±1.5 f for larger loads.

Instead of 0.5 f, 1 f and 1.5 f the values 1 f, 2 f and 3 f can be used analogously.

2.3.2 Digital rounding errors and f

A) Indication with a scale interval d smaller than f

With the aid of f one can decide whether or not the digital rounding errors included in digital indications should be eliminated. Without any special measures, the rounding errors are deemed to be eliminated if

$$f \geq n\,d \quad (n \geq 5)$$

where d is the scale interval of the instrument (OIML R 76-1, T.3.2.2).

B) Changeover point

Otherwise, the so-called changeover point (obtained after adding successive weights of 0.1 d or 0.2 d) is used to determine the "accurate" indication such as is given in OIML R 76-1, A.4.4.3. (The zero indication is dealt with in this way, too (see also 4.5.2 B). The errors recorded in the calibration should be corrected for the error at zero obtained at the start of the weighing test).

2.3.3 Errors of net values

Let us deal with the case where a tare load is placed on the instrument and a tare device is used to set the indication to zero. Thereafter, weighings are performed with the net loads. In order to estimate the errors of the net values we use the weighing curve obtained in the calibration of the instrument. This is done as described below.

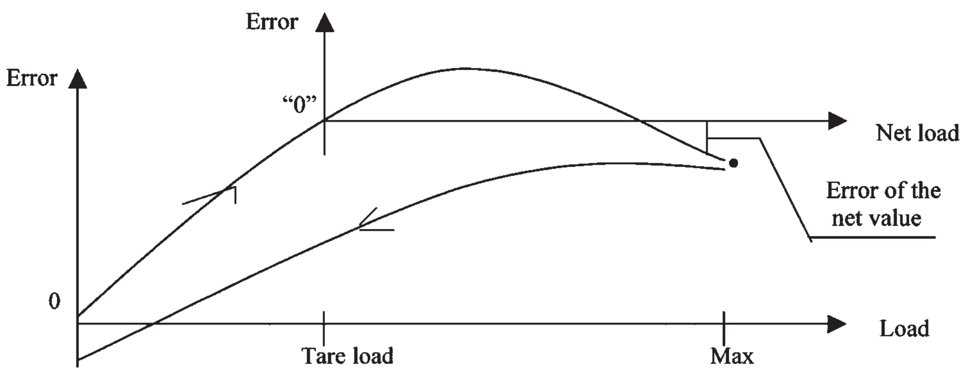

Figure 1 *The (>) branch of the weighing curve is obtained with increasing loads and the (<) branch with decreasing loads. When a tare load is on the instrument a subtractive tare device (reducing the weighing range for the net loads) is used to set the indication to zero "0".*

Let "0" be the zero indication after operation of the tare device. Plot "0" on the weighing curve so that it corresponds to the tare load (see Figure 1). Draw new coordinates through this zero point "0". From the weighing curve the errors of the net values are obtained in the new coordinates.

2.3.4 Standard weights and notes on calibration

A) Selection of weights

For the weighing test the standard weights are selected as follows (OIML R 76-1, 3.7.1).

Verified weights

The sum of the absolute values of the maximum permissible errors (mpe's) of the weights (≤ 50 kg) shall not be greater than 1/3 of the |MTE| of the instrument for the applied load.

Weights ≥ 50 kg, such as 500 kg and 1000 kg, are selected so that Max/f of the instrument is less than or equal to the n marked on the weights.

Calibrated weights

If the indications of the instrument are not corrected for the errors of the weights, the sum of the absolute values of the errors of the weights shall not be greater than 1/3 of the |MTE| of the instrument for the applied load.

If the indications of the instrument are corrected for the errors of the weights, the sum of the absolute values of the uncertainties of the weights shall not be greater than 1/3 of the |MTE| of the instrument for the applied load.

Preferably 1/5 should be used instead of 1/3 above (see 4.2.2, *Note 1*).

B) Notes on calibration

The calibration should take place at the site where the instrument is used.

If the instrument is to be repaired, serviced or its span adjusted, then before these operations, the development of the properties of the instrument should be inspected by performing the weighing test and the test described in 4.5.1. The results should be documented. After the repair, etc. the instrument can be calibrated.

3 Uncertainty of calibration, its nature, practical meaning and evaluation

3.1 "True" E and value of uncertainty

For a certain load let the error (2.1) of the instrument be E and the value of the uncertainty of calibration U. It shall be such that the interval E ± U covers the "true" value of E with a "high" confidence. The "true" value of E is here called the "true" E.

One might think that the larger U is the more confidence could be gained. If U is enlarged deliberately, U can have a "large" confidence, say, " >1 ". However, from the point of view of the measurements the information included in U can even be very poor. On the other hand, if U is rigorously evaluated according to the *GUM*, U has a confidence < 1. It is large enough, and the information is as good as possible. Let us say, U has a physical meaning.

3.2 Nature of uncertainty

The uncertainty of calibration of a weighing instrument consists of different components such as the properties of the instrument and the errors of the standard weights used in the calibration. The components are never completely known. So the uncertainty, which depends on the components, is not completely known either. This means that the value of the uncertainty is a realization of a random variable and hence its value cannot be predetermined.

3.3 Practical meaning of uncertainty

3.3.1 Relationship between |E| ≤ |MTE| and |"true" E| ≤ |MTE|

Let us restrict ourselves to the case where the absolute values of the errors E obtained in the calibration are within the MTE's

|E| ≤ |MTE|

for all the loads. The question is: how well the condition |"true" E| ≤ |MTE| can be estimated from |E| ≤ |MTE| when U takes on different values?

A) U ≤ 1/3 x |MTE|

If |E| ≤ 2/3 x |MTE| and U ≤ 1/3 x |MTE|, then inserting these values of |E| and U in E ± U (which includes the "true" E) it is easy to see that |"true" E| ≤ |MTE| is true.

In general, if |E| ≤ |MTE| and U ≤ 1/3 x |MTE|, the probability P that the condition |"true" E| ≤ |MTE| is true is approximated by the fraction |MTE| / (|MTE| +1/3 x |MTE|). Now |MTE| is half the length of the interval where the "true" E should be and |MTE| + 1/3 x |MTE| that where it is. If U < 1/3 x |MTE|, P is greater than the fraction and if U =1/3 x |MTE|, P equals the fraction. So

P ≥ |MTE| / (|MTE| + 1/3 x |MTE|) = |MTE| / (4/3 x |MTE|) = 75 %

B) U < |MTE|

U = k x |MTE| (k < 1). Such as in *A)* the probability P that the condition |"true" E| ≤ |MTE| is true is

P = |MTE| / (|MTE| + k x |MTE|) = 1 / (1 + k) > 50 % (k < 1)

Example: Let the observed E be E= + 0.4 x|MTE|. If k = 0.9, then the "true" E is in the interval E ± 0.9 x |MTE| (its length is 1.8 x |MTE|). In order for the condition |"true" E| ≤ |MTE| to be true, the "true" E should be in the interval from – 0.5 x |MTE| to |MTE| the length of which is 1.5 x |MTE| (– 0.5 x |MTE| = 0.4 x |MTE| – 0.9 x |MTE|). Thus P = 1.5 x |MTE| / (1.8 x |MTE|) ≈ 83 %.

C) U ≥ |MTE|

U = k x |MTE| (k ≥ 1). The probability P that the condition |"true" E| ≤ |MTE| is true is

P = |MTE| / (|MTE| + k x |MTE|) = 1 / (1 + k) ≤ 50 % (k ≥ 1)

3.3.2 "Suitable" values of U

Here values of U < |MTE| are regarded as suitable, i.e., a probability P > 50 % that the condition |"true" E| ≤ |MTE| is true is preferred. Values of U ≤ 1/3 x |MTE| are ideal but may sometimes be difficult to achieve (see 4.2.2, *Note 1*).

If U ≥ |MTE|, there is a need to take measures to reduce U. It is possible if the instrument is giving service and if errors such as the repeatability and the eccentric errors (4.2 and 4.5.1) of the instrument can be made as small as possible. An alternative is if the values of the |MTE|'s can be increased.

3.4 Methods of evaluation of uncertainty

3.4.1 Method in principle

The presentation of this method serves as an introduction to the evaluation of the uncertainty.

Imagine that the calibration (the weighing test) is repeated at least five times, each time applying the standard weights to the load receptor so as to produce centric and several kinds of eccentric loading. To simplify the presentation the ambient temperature is supposed to be unchanged during the tests. For each load the standard deviation s and the variance s^2 of the errors obtained in the successive tests are calculated. For each different load the variance of the errors of the weights u^2 is combined with the corresponding value of s^2 respectively (u^2 is calculated as described in 4.4 but now for each load separately). The uncertainty is $U = 2(s^2 + u^2)^{1/2}$.

3.4.2 Practical method

In this method, which is described in Section 4 below, the calibration (the weighing test) is performed only once and the different uncertainty components are investigated. With their aid the uncertainty, which is the "tool" for judging the calibration results, is evaluated.

In order to investigate the uncertainty components, different tests are performed, e.g., the repeatability test where several weighings are carried out with the same load, and the eccentricity test where weighings are made with the same load in different positions on the load receptor. The results of these tests are the differences between the results of the weighings. They do not show how they relate to some "true" values. Thus the results are test results and not calibration results.

4 Relationship between uncertainty of calibration and its components

Here the relationship is given in formula (A) for U and (B) for U_1.

The terms in U are related to:

1) the repeatability of the instrument,
2) the rounding errors involved in digital indication if these are not eliminated (2.3.2),
3) the errors in the standard weights used in the weighing test,
4) the errors brought about by possible eccentric positioning of the weights in the weighing test,

5) variations in the zero point, and

6) the effect of temperature variations during the weighing test.

The terms in U_1 are related to those given above and to 7a) variations in the indication due to influences such as vibration and disturbances during the weighing test and 7b) irregularities in the weighing curve.

The uncertainty components are 1) to 7b). The components 1), 2) and 3) are the main ones which always have an influence on U except 2) where rounding errors are eliminated. Components 4), 5) and 6) have a real effect on U only if they are powerful enough. Their combined effect is here denoted by the symbol z. The components 7a) and 7b) are denoted by the common symbol w.

The relationship between U (the uncertainty, w = 0) and its components is:

(A) $U = 2r\,[(k_n R)^2 + 1/r^2\,(0.3\,d)^2 + u^2 + z^2\,]^{1/2}$

The relationship between U_1 (the uncertainty, w ≠ 0) and its components is:

(B) $U_1 = 2r\,[(k_n R)^2 + 1/r^2\,(0.3\,d)^2 + u^2 + z^2 + 1/r^2\,w^2\,]^{1/2} = [U^2 + 4\,w^2]^{1/2}$

2 r, k_n, R, d, u, z and w are explained below in points 4.1 to 4.6 (see also Section 5). U is dealt with in 4.1 to 4.6.1 and U_1 in Table 1 (4.1) and in more detail in 4.6.2.

4.1 Coefficient 2 r and Table 1

2 is the coverage factor (*GUM*). With its aid the values of U meet the condition of the “high confidence” given at the beginning of 3.1.

r is for nonautomatic single-interval instruments. It assumes the values r = 1, 0.7, 0.4 and 0.3 which are determined on the basis of consideration. How r is used is presented below and in Table 1.

Meaning of r

Imagine that the weighing range 0 – Max is divided into parts 1 to 3 in which the MTE’s (2.3.1) assume different values, e.g., ±0.5 f, ±1 f and ±1.5 f. Each part is associated with a value of r. If there is only one part (MTE = ±1 f for all loads), then r = 1. If there are two or three parts, r = 1 for the part with the largest loads and r < 1 for the others. The smaller the loads are in the parts the smaller the values r assumes.

Each part associated with a value of r is also associated with a value of U. For the part with r = 1 let the value of U be $U_{r=1}$. Therefore, the terms in U are determined with some “large” loads and the values of the terms and r = 1 are used to evaluate $U_{r=1}$. For smaller loads the values of U are evaluated with the same values of the terms as above but with a value for r < 1 obtained from Table 1 (see next page).

4.2 Term $k_n R$ brought about by repeatability R

4.2.1 R and standard deviation $k_n R$

The repeatability R is the difference between the largest and the smallest results of the weighings in the repeatability test (4.2.2). The standard deviation of all the results is $k_n R$ where n (n ≥ 3) is the number of the weighings and k_n the coefficient which assumes the following values:

$k_3 = 0.591$, $k_4 = 0.486$, $k_5 = 0.430$, $k_6 = 0.395$, $k_7 = 0.370$, $k_8 = 0.350$, $k_9 = 0.337$ and $k_{10} = 0.325$.

Motivation

The values of k_n are adopted from tables giving parameters of the distribution of the range of samples taken from a normal population (range = the difference between the largest and the smallest observation). The repeatability R is deemed to originate from that population, too. Such a table can be found in: *Hald. A ,“Statistical Tables and Formulas” Willey & Sons, Inc., New York, London, 1952, Table VIII.*

4.2.2 Repeatability test

The “large” test load L_R is $L_R \geq 0.5$ x Max. At least five weighings should be performed with L_R in an identical manner. (Suppose that k_n indicates the quality of information obtained from the repeatability test. It is quite stable if n ≥ 5 (see the values of k_n)). According to OIML R 76-1, A.4.10, the instrument is set to zero before each weighing or the automatic zero-tracking device shall be in operation (in this case the procedure in 2.3.2 B is not applied to digital zero indications).

The rounding errors included in digital indication of the weighings with L_R should be eliminated. If this is not done and if the result R = 0 is obtained, it should be replaced by R = d (the worst case). However, this value of R can result in values of U, which are too large (see 3.1 to 3.3).

Note 1

A) Suppose that the MTE’s take on the values ±0.5 f and ±1 f. If 1) R meets the condition R ≤ 0.35 f and 2) the weights for the weighing test are chosen so that 1/5 is used instead of 1/3 (2.3.4 A) and 3) the other uncertainty components are insignificant, then U ≤ 1/3 x |MTE |.

B) If the MTE’s take on the values ± 0.5 f, ±1 f and ±1.5 f and if R < 0.55 f and 2) and 3) above are met, then U ≤ 1/3 x |MTE |. However, if the weights are chosen so that 1/3 is used, R should be R < 0.4 f and 3) above should be met in order that U ≤ 1/3 x |MTE |.

1) Weighing range divided into three parts		MTE: ± 0.5 f, ± 1 f and ± 1.5 f or ± 1 f, ± 2 f and ± 3 f
Part with MTE = ± 0.5 f or ± 1 f	r = 0.3	$U = 2r\,[(k_n R)^2 + 1/r^2\,(0.3\ d)^2 + u^2 + z^2\,]^{1/2}$
Part with MTE = ± 1 f or ± 2 f	r = 0.7	
Part with MTE = ± 1.5 f or ± 3 f	r = 1	
2) Weighing range divided into two parts		MTE: ± 0.5 f and ± 1 f or ± 1 f and ± 2 f or
Part with MTE = ± 0.5 f or ± 1 f	r =0.4	$U = 2r\,[(k_n R)^2 + 1/r^2\,(0.3\ d)^2 + u^2 + z^2\,]^{1/2}$
Part with MTE = ± 1 f or ± 2 f	r = 1	
3) One part		MTE: ± 1 f for all loads in the weighing range
MTE = ± 1 f	r = 1	$U = 2r\,[(k_n R)^2 + 1/r^2\,(0.3\ d)^2 + u^2 + z^2\,]^{1/2}$

Table 1 Evaluation of U or U_1 for different loads. Single-interval instruments

Suppose that w = 0. For some "large" loads (4.2 to 4.5) determine a value of $k_n R$, u and z (d is the scale interval) in U. These values and r obtained according to the MTE's (Table) are inserted in Formula (A) for U. In this way the values of U (w = 0) can be evaluated for loads at which the MTE's takes on different values.

Suppose that w ≠ 0 for some adjacent loads. For these loads the value of U (w = 0) is corrected for w ≠ 0. The corrected value of U equals the value of $U_1 = [U^2 + 4\ w^2]^{1/2}$ (Formula B). More information on w and U_1 is given in 4.6.2, Figure 2 and Table 2. If w is w ≠ 0 for almost all the loads in the weighing range either U or U_1 is evaluated as described in 4.6.1.

Note 2

With the aid of R, variations in some other quantities which have influence on U can be taken into account. Examples are the variations in the hysteresis and that given in 4.6.1. In order to explain the variations in the hysteresis imagine that the weighing test is repeated under the same conditions. It is likely that the location of the weighing curve of each weighing test is slightly different. For a certain load the changes of the locations will indicate the variations in the hysteresis. It should be emphasized that the hysteresis as such cannot be regarded as an uncertainty component but the variations in it can. However, they can be included in R for all the loads so that with the aid of R the variations in the hysteresis can be taken into account without any other measures.

4.3 Term 0.3 d brought about by digital rounding errors

d is the scale interval. If the rounding errors in the digital indications noted in the weighing test are eliminated (2.3.2), set **$(0.3\ d)^2 = 0$**. Otherwise *0.3 d, which is the standard deviation of the digital rounding errors*, is used to calculate U.

How to obtain 0.3 d

The digital rounding errors are in the interval (–0.5 d, +0.5 d) or more exactly in the intervals kd ± 0.5 d, k = 0,1,2,…, n; n = Max/d, i.e., in the n + 1 intervals (–0.5 d, +0.5 d) around the points kd, k = 0, 1, 2, …, n. The distribution of the rounding errors is uniform (they have the uniform, rectangular distribution). Their standard deviation is 0.5 d/√3 ≈ 0.3 d.

In the formula for U the coefficient of $(0.3\ d)^2$ is $1/r^2$ because the effect of the rounding errors is to be kept unchanged when operating with r.

4.4 Term u brought about by weights

u is the standard deviation of the errors of the weights for a "large" load L_{Rw}. It corresponds to the load L_R used in the repeatability test. The weights in L_{Rw} can be regarded as a representative sample drawn from the weights used in the weighing test. u assumes different values as given below in points 4.4.1 to 4.4.3.

4.4.1 Verified weights

u = 0.4 x (the sum W of the absolute values of the mpe's (2.3.4 A) of the weights in L_{Rw})

Motivation

All the verified weights are here supposed to be adjusted so that their errors are within the limits ± 2/3 x |mpe |. So the actual error of the weights in L_{Rw} can be estimated as being in the interval [–2/3 W, +2/3 W]. The error is uniformly distributed in [–2/3 W, +2/3 W]. (*If there are several weights in L_{Rw}, they may have both positive and negative errors and their sum, the actual error of the weights, can be quite "small". In this case the interval [–2/3 W, +2/3 W] could be an upper estimate of the intervals where the actual error can be found*). The standard deviation of the actual error of the weights is u = 2/3 W/√3 ≈ 0.4 W.

4.4.2 Calibrated weights

A) Two or more weights are used in L_{Rw} and the indications of the instrument are not corrected for the errors of the weights

u = 0.6 x (the sum W of the absolute values of the errors of the weights in L_{Rw})

Motivation

The actual error of the weights in L_{Rw} is estimated to be in the interval [–W, +W]. The distribution of the error is of the discrete type. In this case where two or more weights are used in L_{Rw} the distribution is approximated by the (continuous) uniform distribution. Thus the standard deviation of the error of the weights is u = W/√3 ≈ 0.6 W.

B) Only one weight is used in L_{Rw} and the indications of the instrument are not corrected for the errors of the weights

u = (the absolute value w of the error of the weight in L_{Rw})

Motivation

The error of the weight is either –w or +w. Their standard deviation u is (their mean is 0 and frequency of appearance 0.5) $u = [0.5\,(+w-0)^2 + 0.5\,(-w-0)^2]^{1/2} = w$.

C) The indications of the instrument are corrected for the errors of the weights

u = 0.5 x (the sum of the absolute values of the uncertainties of the weights in L_{Rw})

Motivation

The uncertainty of a weight multiplied by 0.5 is a standard deviation. Let us call it the "standard deviation of the weight". The sum of the absolute values of the uncertainties of several weights multiplied by 0.5 is an upper limit of the joint standard deviation of the corresponding weights.

4.4.3 Effect of air density on weights

If the calibration results of the instruments with Max/f > 50 000 (frequently d ≤ 1 mg) are obtained at different ambient air densities ρ (kg/m^3) which differ from the reference density 1.2 kg/m^3, the results should be corrected for the effect of the air density on the weights.

After the correction the calibration results are equivalent to those which would have been obtained if the air density were 1.2 kg/m^3. The formula for the effect is given in "Motivation" below. Here the corresponding effect on the load receptor and on the load measuring device is not considered.

The uncertainty component due to the correction is $(m/8000)^2\, s_\rho^2$. It is added to u^2. So in the formula for U, u^2 is replaced by

$$u^2 + (m/8000)^2 s_\rho^2$$

where u is that given in 4.4.1 or 4.4.2 and *$m/8000\ s_\rho$ is the standard deviation of the correction of the effect of the air density on the weights*, m (kg) is the mass of the weights in L_{Rw} and s_ρ (kg/m^3) the standard deviation due to the variations of the air density ρ during the weighing test. 8000 kg/m^3 is the standard reference density of the weights.

To obtain ρ the air pressure, the ambient temperature and the relative humidity have to be measured. If the error of the air pressure measured is ± 10 mbar or ± 50 mbar, the value of s_ρ is approximately 0.01 kg/m^3 or 0.04 kg/m^3 respectively. At sites where instruments with Max/f > 50 000 are usually used, the variations of the temperature and the relative humidity can influence the third or successive decimal places in the numerical value of s_ρ.

Motivation

According to Archimedes' Principle the effect of the air density on the weights is obtained from the formula: (1.2 – ρ) m/8000 (kg). From it m/8000 s_ρ is derived.

4.5 Term z brought about by three kinds of errors

The term z includes the following errors: Δ_1, Δ_2 and $\gamma L_R \Delta t$. Δ_1 is brought about by eccentric loadings, Δ_2 by the variations in the zero point and $\gamma L_R \Delta t$ by the temperature variations Δt during the weighing test

$$z^2 = (0.4\ \Delta_1)^2 + 1/r^2\ (0.2\ \Delta_2)^2 + (0.2\ \gamma\ L_R\ \Delta t)^2$$

4.5.1 Largest eccentric error Δ_1

A) Eccentricity test, Δ_1 and standard deviation 0.4 Δ_1

In order to estimate the effect of the weights applied more or less to eccentric positions on the load receptor during the weighing test, the results of the eccentricity test are used here. In this test the same "large" test load (see B below) is successively applied to the eccentric positions (given in OIML R 76-1, A.4.7) and to the middle position of the load receptor. The differences between any indication at the eccentric positions and that at the middle position are determined. The absolute value of the largest difference is Δ_1.

If Δ_1 is smaller than |MTE| (Δ_1 < |MTE|) of the instrument for the load used in the eccentricity test, set **$(0.4\ \Delta_1)^2 = 0$**. Otherwise, Δ_1 is used to calculate z^2. 0.4 Δ_1 is the standard deviation of the eccentric effect of the weights during the weighing test (see C and D below).

B) Test load and corrections

For instruments having not more than four (n ≤ 4) points of support (load cells) the test load is 1/3 x Max. If n > 4, the test load is 1/(n – 1) x Max (OIML R 76-1, A.4.7).

All the indications observed in the eccentricity test should be corrected for the error at zero. This should be determined every time just before the test load is applied to the different positions (WELMEC 2, Directive 90/384/EEC: Common Application, 4.5). This is necessary for instruments with Max / f > 10 000. If the accuracy of the zero-setting is ≤ 0.25 f, the indication of instruments with Max / f ≤ 10 000 can be set to zero before the test load is applied. In addition, the rounding errors included in digital indications should be eliminated.

C) How to obtain 0.4 Δ_1

In order to obtain 0.4 Δ_1 imagine n + 1 weighings. The first n weighings are performed with the load L_R (4.2.2) at the middle position and the last weighing with the same load but applied to an eccentric position on the load receptor. Except for the position of the load for the last weighing, the weighings are performed as in the repeatability test. The joint variance of the results of the weighings is

$$1/n\ \{(n-1)s^2 + [n/(n+1)]\ (x' - m)^2\}$$

where m is the arithmetic mean of the results of the first n weighings, $s \approx k_n R$ (4.2.1) their standard deviation and x' is the result of the last weighing. The components of the joint variance are:

1) $[(n-1)/n]\ s^2 \approx [(n-1)/n]\ (k_n R)^2$
   due to the repeatability and

2) $[1/(n+1)]\ (x' - m)^2$
   due to the eccentric loading.

During the weighing test the weights are applied to the load receptor as in 2.2.2. The magnitude of the effect of the error brought about by the eccentric positions of the weights can be described by the distance from the center of gravity of the weights to the midpoint of the load receptor. The shorter the distance is the smaller the eccentric effect for a given load. Usually the distance is quite short.

If the load L_R (≥ 0.5 x Max) in the last weighing above is placed halfway between the midpoint of the load receptor and the position where Δ_1 is obtained in the eccentricity test, |x' – m| (component 2) above) can approximately take on the value Δ_1 (note that in the eccentricity test Δ_1 is obtained with a load which is "much" smaller than L_R). If $|x' - m| \approx \Delta_1$ and n = 5, then according to 2) $[1/(n+1)]\ (x' - m)^2 \approx [1/(n+1)]\ \Delta_1^2 \approx (0.4\ \Delta_1)^2$. If the load L_R were placed anywhere else on the halfway point between the midpoint of the load receptor and the positions used in the eccentricity test, |x' – m| could take on a value smaller than Δ_1 or it could even be zero.

D) Effect of $(0.4\ \Delta_1)^2$

If the result Δ_1 of the eccentricity test assumes a "large" value ($\geq |MTE|$) for the applied load, then $(0.4\ \Delta_1)^2$ could have a significant effect on U, i.e., the results of the weighing test are "highly" dependent on the positions of the weights on the load receptor. On the other hand, if Δ_1 assumes a "small" value ($< |MTE|$), the results of the weighing test are almost independent of the positions of the weights and thus, one could say that the effect of $(0.4\ \Delta_1)^2$ on U is negligible.

4.5.2 Difference Δ_2 between zero points

A) Δ_2 and standard deviation $0.2\Delta_2$

Δ_2 is the absolute value of the difference between the zero indications at the start and at the end of the weighing test. If Δ_2 is smaller than $|MTE|$ ($\Delta_2 < |MTE|$) for the zero point, set $\mathbf{(0.2\ \Delta_2)^2 = 0}$. Otherwise Δ_2 is used to calculate z^2. The standard deviation of the variations in the zero point during the weighing test is approximated here with the aid of $0.2\ \Delta_2$.

Motivation

Assume that Δ_2 consists of the hysteresis and the variations in the zero point. The width of the range of the "pure" variations (without the hysteresis and momentary variations in the zero point) is guessed to be $0.7\ \Delta_2$ Assuming that the variations in the zero point are independent and uniformly distributed over $0.7\Delta_2$, the standard deviation of the variations is roughly $0.5 \times 0.7\ \Delta_2 / \sqrt{3} \approx 0.2\Delta_2$.

The coefficient $1/\ r^2$ of $(0.2\ \Delta_2)^2$ in z^2 is to keep $(0.2\ \Delta_2)^2$ unchanged when operating with r.

B) Determination of Δ_2 when zero-tracking is in operation

If the zero-tracking device is in operation, then with a "small" load which is out of the automatic range of the zero-tracking the indication is noted, and its error is defined to be the error at zero (OIML R 76-1, A.4.2.3.2). This is used to determine Δ_2 in connection with the weighing test.

4.5.3 Temperature effect and error $\gamma\, L_R\, \Delta t$

A) $\gamma\, L_R\, \Delta t$ and standard deviation $0.2\ \gamma\, L_R\, \Delta t$

In $\gamma\, L_R\, \Delta t$ the symbol Δt denotes the difference between the extreme temperatures during the weighing test. L_R is the load for the repeatability test (4.2.2) and γ a coefficient so that $\gamma\, L_R\, \Delta t$ is the error of the instrument due to Δt for the "large" load L_R.

If Δt meets the conditions for steady ambient temperature*), set $\mathbf{(0.2\ \gamma\, L_R\, \Delta t)^2 = 0}$. Otherwise, $0.2\ \gamma\, L_R\, \Delta t$ is used to calculate z^2 taking into account the restrictions in B) below. $0.2\ \gamma\, L_R\, \Delta t$ is the standard deviation of the error of the instrument for the load L_R caused by Δt.

How to obtain $0.2\ \gamma\, L_R\, \Delta t$

$0.2\ \gamma\, L_R\, \Delta t$ is derived by regarding $\gamma\, \Delta t$ as a sum of independent "impulses" $\gamma\, \Delta t = \sum \gamma\, \Delta t_k$ (k=1, 2, … , n). Because n can be regarded as a large number, suppose that the conditions of the Central Limit Theorem (see textbooks of statistics) are met for $\gamma\, \Delta t$ and thus it is approximately normally distributed. Then the standard deviation of $\gamma\, \Delta t$ is $1/6 \times \gamma\, \Delta t \approx 0.2\ \gamma\, \Delta t$ (the "observed" sum of the impulses is from zero to $\gamma\, \Delta t$). $0.2\ \gamma\, L_R\, \Delta t$ is obtained by multiplying $0.2\ \gamma\, \Delta t$ by L_R.

B) Δt exceeds limits for steady ambient temperature

If Δt exceeds the limits for the steady ambient temperature*) and if the test has lasted, say, for more than 0.5 h, $0.2\ \gamma\, L_R\, \Delta t$ is deemed to have a significant effect on z^2 and thus on U. (A weighing instrument using electric power is a heat dissipating device with a housing. The effect of the changes of the ambient temperature on the instrument cannot be regarded as significant until after a sufficient delay).

4.6 Term w due to variations in indication and irregularities in weighing curve

4.6.1 Variations in indication and irregularities in weighing curve for several loads

The variations can be brought about by influences such as vibration, draughts, strongly oscillating ambient temperature and electrical disturbances. During the variations an unambiguous reading of the indication may not be possible and thus the variations can have an influence on the uncertainty.

The irregularities in the weighing curve can be due to automatic corrections of properties of the instrument. Due to the irregularities the weighing curve may have a zigzag form and the points of the curve (the errors of the instrument) can be difficult to determine accurately enough. Thus, the irregularities could be a source of the uncertainty.

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*) According to OIML R 76-1, A.4.1.2, Δt should be at most 5 °C for instruments for industrial weighing (Max/f ≤ 10 000), 3 °C for laboratory scales (high accuracy instruments, Max/f ≤ 100 000) and 1 °C for instruments of special accuracy (Max/f unlimited). The rate of change of Δt must not be more that 5 °C /h.

Suppose that due to the variations/irregularities, the repeatability R of the instrument is considerably enlarged. In this case the effect of the variations/irregularities on the uncertainty U can be obtained with the aid of R without any other measures (see 4.2.2, *Note 2*). In this case w is deemed to be **w = 0** and the values of U (w = 0) (Formula A) are determined. However, if the effect of the variations/irregularities is not included in R, w ≠ 0 and the values of U_1 (Formula B) have to be determined for all the loads. The calculation of w is explained in 4.6.2

4.6.2 Variations in indication and irregularities in weighing curve for certain loads only

A) Range Δ_3 and standard deviation w = 0.3 Δ_3

If the variations in the indication or the irregularities in the weighing curve are noted for some adjacent loads, one range Δ_3 of the variations or of the irregularities is estimated for these loads. For example, if the variations are within ±2d, then Δ_3 = 4d, or if the difference between the limits of the "oscillations" in the weighing curve is Δ_3, then Δ_3 is the range of the irregularities. In the case mentioned at the end of 4.6.1, one value of Δ_3 should be estimated for the variations/irregularities for all the loads.

w = 0.3 Δ_3 is the standard deviation of the variations/irregularities. If Δ_3 < 1/3 x |MTE|, set **w = 0**. Otherwise, if $\Delta_3 \geq$ 1/3 x |MTE|, the uncertainty U (w = 0) should be corrected for w ≠ 0 for the loads at which w exists. This results in the uncertainty U_1 (Formula B). The evaluation of U_1 is elucidated in more detail in Figure 2 and Table 2.

Motivation

Suppose that the variations or the irregularities are uniformly distributed over Δ_3. Thus, their standard deviation is w = 0.5 x Δ_3 /√3 ≈0.3 Δ_3.

In the formula of U_1 (Formula B) there is the coefficient $1/r^2$ of w^2. This is because w shall be kept unchanged when operating with r.

B) Nature of U_1

Frequently, the value of U_1 is quite large due to w. For the loads at which w exists it is not possible to perform the calibration accurately enough. To some extent the large values of U_1 indicate inadequate protection of the instrument against the influences and factors causing variations and irregularities respectively. U_1 is not suitable for the investigations presented in 3.3.

The user of the instrument should pay attention to the existence of the variations or irregularities during practical weighings. They have to be taken into account in the uncertainty of weighing. This is briefly dealt with in Section 6.

5 Use of U and examples

5.1 Use of U

The same value of U is associated with the two calibration results obtained for a certain load L. One is obtained when the load L is reached by the increasing and the other by the decreasing loads. This is also adapted for use of U_1, if applicable (see Table 2). In order to judge the calibration results, U is used such as is explained in Section 3.

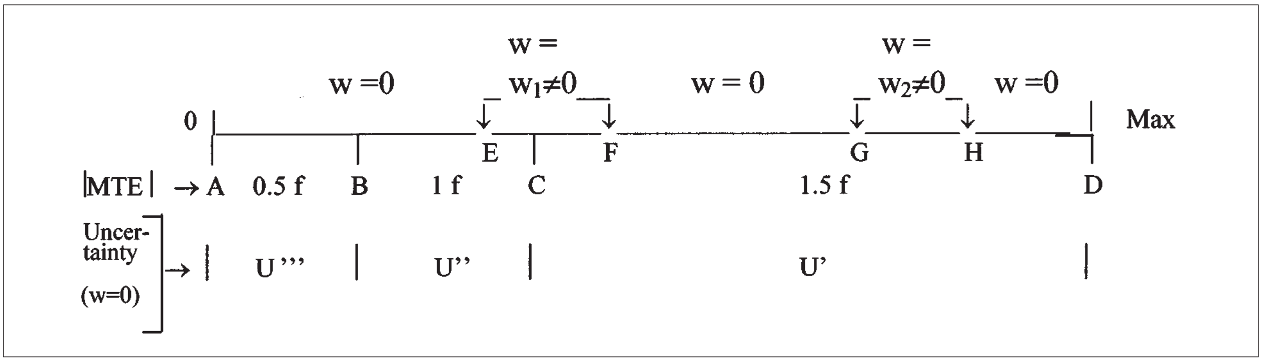

Figure 2 The weighing range 0 – Max is divided into the parts A to B, B to C and C to D. For them the values of the |MTE|'s 0.5 f, 1 f and 1.5 f and the uncertainties U''', U'' and U' (w = 0) are given respectively. However, w ≠ 0 is w_1 for loads from E to F and w_2 from G to H. For these loads the values of the uncertainties U'' and U' should be corrected for w_1 and w_2 respectively, i.e., the values of U_1 (Formula B) are determined. For loads from E to C, $U_1 = [U''^2 + 4w_1^2]^{1/2}$ and from C to F, $U_1 = [U'^2 + 4w_1^2]^{1/2}$. For loads from G to H, $U_1 = [U'^2 + 4 w_2^2]^{1/2}$.

Loads:	A to B	B to E	E to C	C to F	F to G	G to H	H to D
Uncertainty:	U'''	U''	$[U''^2 + 4w_1^2]^{1/2}$	$[U'^2 + 4w_1^2]^{1/2}$	U'	$[U'^2 + 4w_2^2]^{1/2}$ *	U'

Table 2 Summary of values of uncertainties in Figure 2

Assume that w_1 exists both for increasing (E to F) and for decreasing loads (F to E), but for loads from G to H w_2 exists for decreasing loads (H to G) only.

* The uncertainty $U_1 = [U'^2 + 4\,w_2^2]^{1/2}$ has to be associated with the calibration results which are obtained for decreasing loads H to G. For increasing loads G to H the uncertainty is U' (see also 5.1).

5.2 Examples

5.2.1 Example 1

In this example the calibration of a vehicle instrument with digital indication is dealt with. The instrument has four load cells, its Max is 20 000 kg and d = 10 kg.

The |MTE|'s for the instrument are f = 10 kg for loads from 0 to ≤ 5 000 kg and 2 f = 20 kg for loads over 5 000 kg to Max.

For the calibration nine verified weights of 2 000 kg, one of 1 000 kg and two of 500 kg are used. They bear the marking n = 3 000, i.e., their maximum permissible errors (mpe's) are ±340 g, ±170 g and ±85 g respectively.

A) Tests (Contrary to recommendations in 2.3.2 and Section 4 all the following tests are performed without eliminating the digital rounding errors)

Weighing test (2.2):

Both for increasing and decreasing loads the observed errors E are 0 for loads from 0 to ≤ 5 000 kg and +10 kg for the loads over 5 000 kg.

Repeatability test (4.2):

A vehicle of about 19 000 kg mass is used as the test load L_R. Six weighings are performed. Thus $k_n = k_6 \approx 0.4$. The observed result is R = 0 but according to 4.2.2 R = d = 10 kg has to be used. So k_6 R = 4 kg.

Weights (4.4):

For the load L_{Rw} = 19 000 kg the sum W of the absolute values of the mpe's of the weights is W = 9 x 340 g + 1 x 170 kg = 3 230 g. According to 4.4.1 u = 0.4 x 3230 g ≈ 1.3 kg.

Eccentricity test (4.5.1), zero return (4.5.2) and temperature effect (4.5.3):

The eccentricity test is performed with a load 6 500 kg (≈1/3 x Max). Δ_1 = 10 kg (< 2f = 20 kg). Δ_2 = 0 kg (< f = 10 kg) and the slow changes Δt in the temperature were about 2 °C during the weighing test. On this basis set $z^2 = (0.4\,\Delta_1)^2 + 1/r^2\,(0.2\,\Delta_2)^2 + (0.2\gamma\,L_R\,\Delta t)^2 = 0$.

During the weighing test variations in the indication were not noted and thus w = 0 (4.6).

B) Evaluation of U (w = 0)

Denote z^2 = 0. For loads from 0 to ≤ 5 000 kg the uncertainty U is (Table 1, point 2, **r = 0.4**):

$U = 2r\,[(k_n\,R)^2 + 1/r^2\,(0.3\,d)^2 + u^2]^{1/2} = 2 \times 0.4 \times [4^2 + 1/0.4^2\,(0.3 \times 10)^2 + 1.3^2]^{1/2} \approx 7$ kg

For the loads > 5 000 kg U is (Table 1, point 2, **r = 1**):

$U = 2r\,[(k_n\,R)^2 + 1/r^2\,(0.3\,d)^2 + u^2\,]^{1/2} = 2 \times [4^2 + (0.3 \times 10)^2 + 1.3^2]^{1/2} \approx 10$ kg

C) Conclusions

Loads from 0 to ≤ 5 000 kg, |MTE| = 10 kg and U = 7 kg

The observed errors E are E = 0 but actually they are in the range of –5 kg to +5 kg. The effect of this range is taken into account by using the term $(0.3\ d)^2$ in U. E ± U = (0 ± 7 kg) and is within the MTEs for the loads in question. So the values of the "true" E satisfy the condition |"true" E| ≤ |MTE|. The probability P (3.3.1) is 100 %.

Loads > 5 000 kg, |MTE| = 20 kg and U = 10 kg

The observed errors E are E= +10 kg but actually they are in the range of +5 kg to +15 kg. This is taken into account by using the term $(0.3\ d)^2$ in U. E ± U = (10 ± 10 kg) and is within the MTEs for the loads in question. So the values of the "true" E satisfy the condition |"true" E| ≤ |MTE|. The probability P is 100 %.

5.2.2 Example 2

In this example the calibration of a laboratory scale (high accuracy instrument) with digital indication is dealt with. The instrument has one load cell, its Max is 12 000 g and d = 1 g.

The |MTE| for the instrument is f = 2 g for all the loads from 0 to Max.

Verified weights 0.5 kg, 1 kg, 2 kg, 5 kg and 10 kg are used. Their OIML accuracy class is M_1.

A) Tests (According to the recommendations in 2.3.2 and in Section 4 all the following tests are performed eliminating the digital rounding errors)

Weighing test (2.2):

Both for increasing and decreasing loads the observed errors E are monotonically increasing from 0 to + 1.5 g for the loads from 0 to Max respectively.

Repeatability test (4.2):

The 10 kg weight is used as the test load L_R. Six weighings are made. R = 0.6 g and $k_n = k_6 \approx 0.4$. Thus k_6 R = 0.24 g.

Weights (4.4):

L_{Rw} = 10 kg. The mpe of the 10 kg weight is 500 mg. Thus W = 500 mg. According to 4.4.1 u = 0.4 x 500 mg ≈ 0.2 g.

Eccentricity test (4.5.1), zero return (4.5.2) and temperature effect (4.5.3):

The eccentricity test is performed with a 4 kg load (= 1/3 x Max). Since Δ_1 = 2 g (= f = 2 g) <u>it is used to calculate z^2</u>. Δ_2 = 0.2 g (< f = 2 g) and the slow change Δt in the temperature was about 1 °C during the weighing test. On this basis $z^2 = (0.4\Delta_1)^2 + 1/r^2(0.2\Delta_2)^2 + (0.2\gamma L_R \Delta t)^2 = (0.4\,\Delta_1)^2 = 0.8^2$.

During the weighing test no variations in the indication were noted and thus w = 0 (4.6).

B) Evaluation of U (w = 0)

Denote $(0.3\text{ d})^2 = 0$. For all the loads from 0 to Max the uncertainty U is (Table 1, point 3, **r = 1**):

$U = 2r\,[(k_n\,R)^2 + u^2 + z^2]^{1/2} = 2 \times [0.24^2 + 0.2^2 + 0.8^2]^{1/2} \approx$ 1.7 g

C) Conclusions

The observed errors E are noted to be from 0 to + 1.5 g. Thus E ± U can take on values from (0 ± 1.7) g to (1.5 ± 1.7) g. For the loads near zero, the value of the positive "true" E is from 0 to 1.7 g, and at Max it is from 0 to 3.2 g. The positive "true" E should be in the interval 0 to 2 g. For the loads near zero P is P = 100 % but for the Max load P is 2 g/3.2 g ≈ 62 % but increases as the loads decrease.

However, if the eccentric error Δ_1 were adjusted so that Δ_1 < 2 g, its effect could be neglected and the uncertainty would be $U = 2r\,[(k_n\,R)^2 + u^2]^{1/2} = 2 \times [0.24^2 + 0.2^2]^{1/2} \approx 0.6$ g. Except for the Max load P is now 100 %.

6 Uncertainty of practical weighing

Suppose that a calibrated instrument is used. The uncertainty of its calibration results is now considered to be one part of the uncertainty of practical weighing with the instrument.

Another part can be obtained by performing weighings with a large load in real weighing situations. Either 1) the weighings are made with the same large load several times or 2) with different large loads the results of which are checked with a control instrument.

In case 1) calculate the variance s^2 of the results the number n of which is n ≥ 5. The variance s^2 can also be determined by means of the range of the results. Therefore, choose k_n according to 4.2.1 and calculate $s^2 \approx (k_n \text{ x range})^2$.

In case 2) calculate the variance v^2 of the differences between the results of the loads obtained with the instrument in question and with the control instrument. The number n of the loads weighed should be n ≥ 5. v^2 can also be calculated with the aid of the range of the differences.

Both s^2 and v^2 are assumed to be large compared with $(k_nR)^2$ obtained in the repeatability test of the instrument (4.2).

The uncertainty of weighing is approximated by the combination of the uncertainty of calibration and the variance s^2 or v^2. The combination is:

$$(U^2 + 4\,s^2)^{1/2} \text{ or } (U^2 + 4\,v^2)^{1/2}$$

where U (w = 0) is the uncertainty of calibration with large loads for which r = 1 (4.1). For smaller loads the uncertainty of weighing should be estimated in the same way taking into account r < 1 in U for these loads.

If w in 4.6.2 exists during the weighings, its value is determined as explained in 4.6.2 and $U_1 = (U^2 + 4\,w^2)^{1/2}$ is calculated. This U_1 is used instead of U in the above formulae of the uncertainty of weighing. Note that if w is w ≠ 0 during the calibration neither it nor the corresponding U_1 for the calibration are used in the uncertainty of weighing. ■