UNCERTAINTY

Role of measurement uncertainty in deciding conformance in legal metrology(*)

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Abstract

The method used to decide whether an instrument conforms with the requirements for legal metrology has an important impact on the accuracy that can be subsequently achieved. There are two approaches to deciding on conformity, the classical approach that does not take uncertainty directly into account, and a more modern approach that is consistent with the industrial decision rules for proving conformity with specifications.

On the basis of a consistent mathematical treatment, the consequences of using the different approaches are demonstrated, along with their influence on the uncertainty contribution of verified instruments that are being used.

Introduction

The accuracy of measuring instruments must be consistent with their intended use. ISO 9001: 2000 and ISO / IEC 17025: 2000 standards [1] [2], require that traceability of measuring and test results to national or international standards must be given in order to allow the necessary statements about their metrological quality. The most important methodologies used to ensure that measuring instruments are giving the correct indication are:

- In industrial metrology: regular calibration of the measuring instruments according to the quality system in use; and
- In legal metrology: type testing and periodic verifications of the measuring instruments according to legal regulations.

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Both methodologies are closely related and are based substantially on the same measuring procedures. Over the years, however, they have become established with separate rules and metrological infrastructures, and they aim at different areas of application.

Legal verification of the conformity of measuring instruments is a method of testing covered by legal regulations. It is part of a process of legal metrological control that in many economies requires type evaluation and approval of some types of instruments as a first step.

However, the use of legally verified instruments within the framework of quality management sometimes presents problems, since only the maximum permissible errors (MPE) for the instruments are stated, without the measurement uncertainties being explicitly given. The relationship of legally prescribed error limits and measurement uncertainty is insufficiently understood. The most important concern for the instrument user therefore is the equivalence and relationship of measurement results which have been obtained from verified and from calibrated instruments.

In order to answer this concern, the understanding of the role of measurement uncertainty in deciding conformity plays a central role, along with the estimation of the uncertainty contributions of verified or conformity tested instruments when they are being used.

Verification and measurement uncertainty

Constituents of legal conformity verification

The constituents are:

- Qualitative tests, predominantly for the state of the instrument and the applicable safety requirements;
- Quantitative tests which are consistent with the definition of calibration (see VIM 6.11 [3]);
- Evaluation of the results of the qualitative and quantitative tests to ensure that the legal requirements are being met; and
- If the evaluation leads to the instrument being accepted: placing a verification mark on the instrument, and, on request, issuing a certificate.

Measurement uncertainty associated with the results of the quantitative tests

The aim of the quantitative tests is to determine the instrumental errors together with the associated uncertainty of measurement at prescribed testing values. The tests are carried out according to well-established and standardized testing procedures. These procedures are mostly identical to those which are used for calibration in industrial metrology. Following the definition of calibration (see VIM 6.11 [3]), a quantitative test may be considered a calibration. Comparison methods are predominantly used for these tests.

Figure 1 shows the block diagram of a typical comparison of an instrument under test and a standard which, in the given example, is a material measure [3]. The standard reproduces or supplies known values of the measurand X_S.

From the block diagram, the measurement error ΔX of the instrument under test may be described by the equation:

$$\Delta X = X_{INDX} - X_S - \delta X_{CS} - \delta X_P \quad (1)$$

δX_{CS} is the unknown error of the standard due to an imperfect calibration of the standard itself;

X_{INDX} is the indication of the instrument under test;

δX_P may be the combination of all other unknown measurement errors due to imperfections of the measuring procedure and of the instrument under test.

$$\delta X_P = \delta X_{DS} + \delta X_{PS} + \delta X_{CPL} + \delta X_{PX} + \delta X_{INDX} \quad (2)$$

Where:

δX_{DS} is the unknown error of the standard due to drift effects;

δX_{PS} is the unknown error of the standard due to its susceptibility to the (incompletely known) environmental conditions;

δX_{CPL} is the unknown error due to the imperfect coupling of the measurand with the instrument under test, e.g. caused by temperature difference, pressure loss, electrical mismatch, etc.;

δX_{PX} is the unknown error due to the imperfection of the instrument under test and its susceptibility to the (incompletely known) environmental conditions;

δX_{INDX} is the unknown error due to the digital resolution or the need to estimate an analogue reading.

The expectation of the measurement error $E[\Delta X] = \Delta x$ is:

$$\Delta x = E[X_{INDX}] - E[X_S] - E[\delta X_{CS}] - E[\delta X_P] \quad (3)$$

where the capital E symbolizes the expectation value of the respective quantity in brackets.

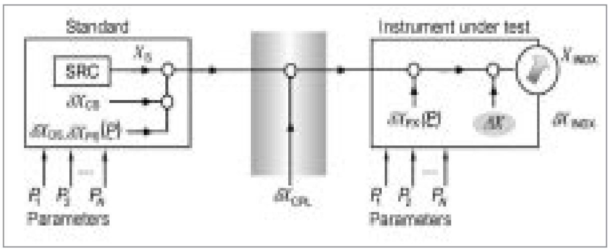

Fig. 1 Comparison method for quantitative testing and calibration using a material measure [3] as a standard. SRC - source of the quantity X_S; other quantities - see text

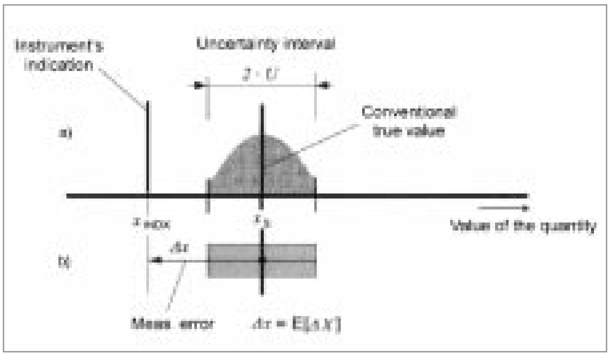

Fig. 2 Calibration result: Illustration of the relationship between the expected value of the measurement error, Δx, and the associated (expanded) uncertainty, U, $U = k \cdot u(\Delta x)$ [5]

a) when stating the conventional true value together with the indicated value,

b) when stating the conventional true value or the indicated value together with the error Δx.

Assuming that all quantities are independent, the square of the standard uncertainty associated with the expectation value of the measurement error can be calculated by:

$$u^2(\Delta x) = u^2(\delta x_{CS}) + u^2(\delta x_{DS}) + u^2(\delta x_{PS}) + u^2(\delta x_{CPL}) + u^2(\delta x_{PX}) + u^2(\delta x_{INDX}) \quad (4)$$

The uncertainty contribution $u(\delta x_{CS})$ can be derived from the uncertainty statement given on the calibration certificate of the standard, and the contribution $u(\delta x_{DS})$ from the existing knowledge about its long-term stability. All other contributions can be estimated from the knowledge about the quantitative test or calibration.

Figure 2 illustrates the relationship between the expected value of the measurement error and the associated (expanded) uncertainty of measurement U when presenting a (single) calibration result.

Equation (4) demonstrates the key problems associated with calibrations:

- The result is valid only for the moment of calibration.
- The result is valid only for the specific calibration conditions.

- The result and, therefore, the quality of dissemination of a physical unit, depend on the performance of the individual instrument under test.

It must be accepted that instruments are often used in environments that are different from the calibration or test conditions.

Therefore, the measurement uncertainty that has been evaluated for laboratory conditions will often be exceeded if the instrument is susceptible to environmental influences. A problem can also arise if the instrument's performance degrades with prolonged use. The instrument user must, therefore, consider all these problems on the basis of his technical knowledge.

Assessment of compliance in legal metrology

Specification limits and uncertainty of measurement

If an instrument is tested for conformity with a given specification or to check that it meets a requirement with regard to error limits, this test consists of comparisons of the calibration results, that give the measurement errors, with the specified values and limits respectively. The uncertainty of measurement associated with the calibration result (see Fig. 2 and equation (4)) inevitably then becomes an uncertainty of the conformity decision. Measurement results affected by measurement errors lying close to prescribed error limits, MPE_- and MPE_+, cannot definitely be regarded as being, or not being, in conformance with the given tolerance requirement. Figure 3 (taken from the standard ISO 14253-1 [4]) makes this problem quite clear: apparently, between the conformance zone and the upper and lower nonconformance zones there are uncertainty intervals that are also called zones of ambiguity. The uncertainty intervals are defined by:

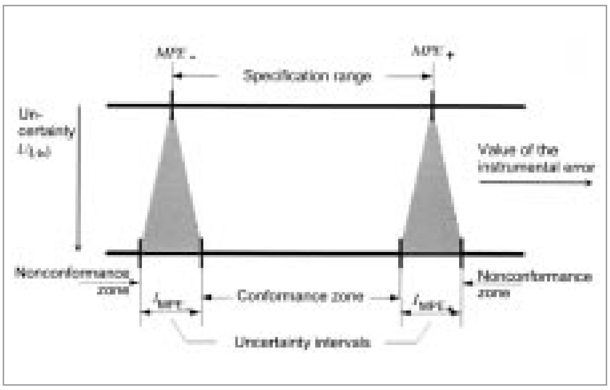

Fig. 3 Specification and measurement uncertainty, $U(\Delta x)$, which is associated with the value of the measurement error, Δx, according to ISO 14253-1 [4].

I_{MPE-} and I_{MPE+} are the lower / upper uncertainty intervals (see text)

$I_{MPE-} = [MPE_- - U; MPE_- + U]$ and

$I_{MPE+} = [MPE_+ - U; MPE_+ + U]$.

According to the explanation of the expanded uncertainty of measurement given in the *Guide to the expression of uncertainty in measurement* (GUM) [5], it can be expected that values lying outside the uncertainty intervals, can be assigned with a high probability, either to the conformance or to the nonconformance zones. When instruments are bought and sold, this conclusion forms the basis for demonstrating conformity or nonconformity.

Decision criteria

Classical approach of legal metrology

The classical approach of legal verification does not take measurement uncertainty directly into consideration. Measuring instruments are normally considered to comply with the MPE requirement if they meet the following criteria:

(a) The value of the instrumental error of the instrument under test is found to be equal to or less than the value of the prescribed maximum permissible error on verification (MPE):

$$|\Delta x| \le MPE \tag{5}$$

(b) The expanded uncertainty of measurement associated with the value of the measurement error, for a coverage probability of 95 %, is small compared with the legally prescribed error limits.

In verification, the expanded uncertainty of measurement U_{95} is usually considered to be small enough if it does not exceed 1/3 of the value of the respective error limit:

$$U_{95} \le U_{max} = 1/3 \cdot MPE \tag{6}$$

where U_{max} is the maximum acceptable value of the expanded uncertainty of measurement associated with the value of the measurement error.

On type testing, the maximum acceptable value of the expanded uncertainty of measurement is reduced to:

$$U_{95type} \le U_{maxtype} = 1/5 \cdot MPE \tag{6a}$$

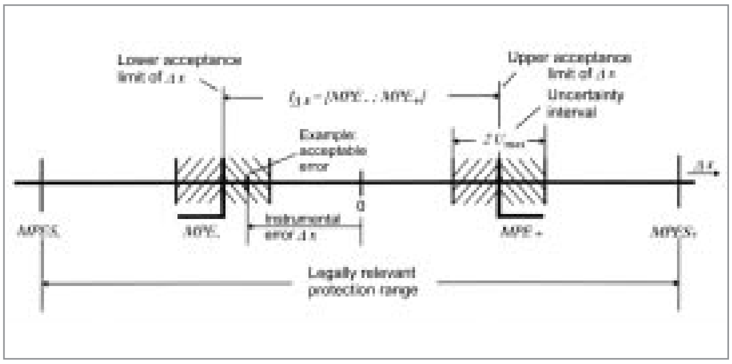

Fig. 4 Illustration of the decision criteria according to the classical verification approach.
MPE_- and MPE_+ are the lower / upper maximum permissible errors on verification;
$MPES_-$ and $MPES_+$ are the lower / upper maximum permissible errors in service;
Δx value of the instrumental error;
$I_{\Delta x}$ error acceptance interval;
U_{max} see equation (6)

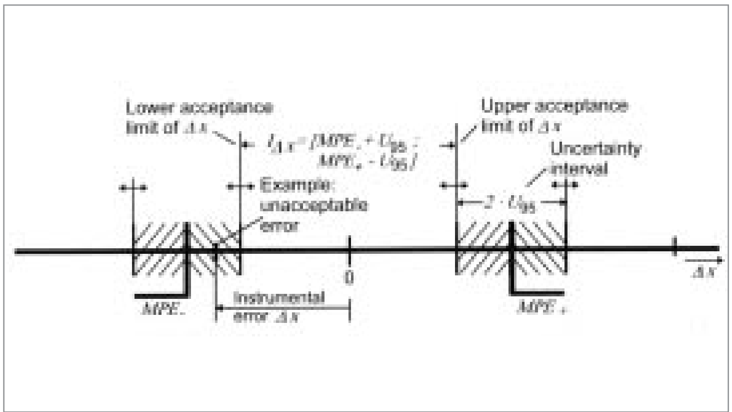

Fig. 5 Illustration of the decision criteria according to the modern approach of evaluating conformity.
MPE_- and MPE_+ are the lower / upper maximum permissible errors on verification;
Δx value of the instrumental error;
$I_{\Delta x}$ error acceptance interval;
U_{95} actual expanded uncertainty of measurement associated with Δx

The decision criteria for verification are illustrated in Fig. 4. The legally prescribed error limits, MPE_- and MPE_+, are equal to the acceptance limits of the instrumental error Δx.

Because of the associated uncertainty, which may extend up to the value U_{max}, it can be expected that, in the worst case, the given error limits on verification will be exceeded by the value of U_{max}, i.e. by 33 % (see equation (6)).

It should be noted that in many economies with developed legal metrology systems, a second kind of error limits has been defined: the maximum permissible errors in service (MPES). These are normally twice the maximum permissible errors on verification. For the instrument user, the maximum permissible errors in service are the error limits that are legally relevant [6]. Therefore, there is only a negligible risk in the sense that no measured value under verification, even if the measurement uncertainty is taken into account, will be outside the tolerance band which is given by the error limits in service (see Fig. 4).

Modern approach to deciding on conformity

In today's metrology, another approach is widely used too. In the regulated area, it is applied to testing of working standards, e.g. weights [7]. This approach is consistent with the prescribed procedures for statements of conformance of calibration results in industrial metrology [8] and with the decision rules given to ISO 14253-1 [4].

Here, instruments are considered to comply with a given specification or with the legal requirements for error limits if they meet the following criteria:

(a) The value of the instrumental error Δx of the instrument under test is found to be equal to or less than the difference between the value of the prescribed error limits, MPE, and the actual expanded uncertainty of measurement, U_{95}:

$$|\Delta x| \leq MPE - U_{95} \tag{7}$$

where U_{95} is the actual expanded uncertainty of measurement associated with the value of the instrumental error Δx.

(b) The expanded uncertainty of measurement associated with the value of the instrumental error, for a coverage probability of 95 %, is small compared with the prescribed error limits.

When verifying weights [7], the expanded uncertainty of measurement, U_{95}, is usually considered to be small enough if it does not exceed 1/3 of the respective error limit. Therefore, equation (6) also applies.

In practice, this means that with respect to measurement errors, Δx, an acceptance interval is defined that is significantly reduced when compared with the range between the prescribed error limits. The magnitude of this interval may be defined by:

$[MPE_- + U_{95}; MPE_+ - U_{95}]$.

This approach is illustrated in Fig. 5.

This approach ensures that there is a high probability that the prescribed error limits are hardly ever exceeded. But, when compared with the classical approach of legal metrology, its practical result is a reduction in the given error limits. Due to the commercial impact of such a *de-facto* reduction, common use in legal metrology seems to be unlikely.

Furthermore, it should be noted that, according to equation (7), the acceptance limits of the error value Δx depend on the value obtained for the expanded uncertainty U_{95} by the performing laboratory. This means that the acceptance limits are not constant, but may vary depending on the competence of the laboratory.

Use of legally verified instruments

In practice, it is often necessary or desirable to determine the uncertainty of measurements that are carried out using legally verified instruments.

The uncertainty of measurement attributed to the measurand is to be estimated according to the GUM [5]. Figure 6 shows the block diagram of a typical direct measurement for which the following equation can be derived:

$$Y = X_{\mathrm{IND}} - \delta X_{\mathrm{M}} - X_{\mathrm{Delta}} \qquad (8)$$

Where:

Y is the measurand, X_{IND} the indication of the measuring instrument;

δX_{M} represents a combined unknown measurement error that comprises all unknown measurement errors due to the imperfection of the measurement procedure and of the measuring instrument in use; and

X_{Delta} is the output quantity either from the instrument's verification or from a calibration.

As an aid to understanding, the uncertainty contribution of a calibrated instrument may first be evaluated. In this case, the output quantity X_{Delta} of the previous calibration of the instrument is the measurement error, and equation (8) becomes:

$$Y = X_{\mathrm{IND}} - \delta X_{\mathrm{M}} - \Delta X \qquad (8a)$$

δX_{M} comprises the result of at least the following error sources (see Fig. 6):

- δX_{PM} the susceptibility of the instrument to environmental conditions and incomplete knowledge of the actual operating conditions;
- δX_{DM} instrument drift;
- δX_{CPLY} imperfect coupling of the measurand to the instrument; and
- δX_{INDM} digital resolution or errors in reading the indication.

From equation (8a), the expectation value of the measurand becomes:

$$y = \mathrm{E}[Y] = \mathrm{E}[X_{\mathrm{INDM}}] - \mathrm{E}[\Delta X] - \mathrm{E}[\delta X_{\mathrm{M}}] \qquad (9)$$

The following standard uncertainty may be attributed to the value of the measurand:

$$u(y) = \sqrt{u^2(\delta x_{\mathrm{M}}) + u^2(\Delta x)} \qquad (10)$$

Both contributions can be assumed to be independent of each other. The contribution $u(\Delta x)$ and the value Δx are known from the result of the previous calibration. The contribution $u(\delta x_{\mathrm{M}})$ must be estimated on the basis of existing knowledge about the measurement.

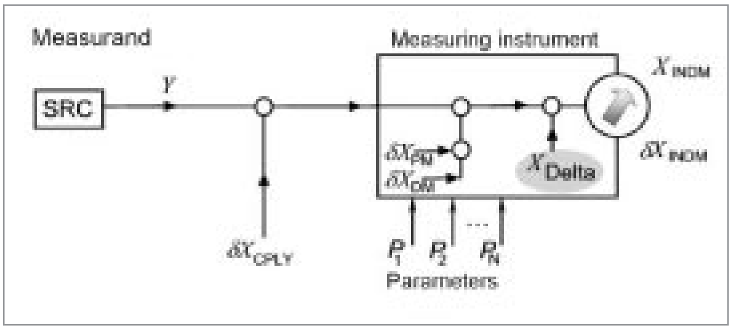

Fig. 6 Direct measurement of the quantity Y (measurand). SRC is the source of the measurand; other quantities - see text

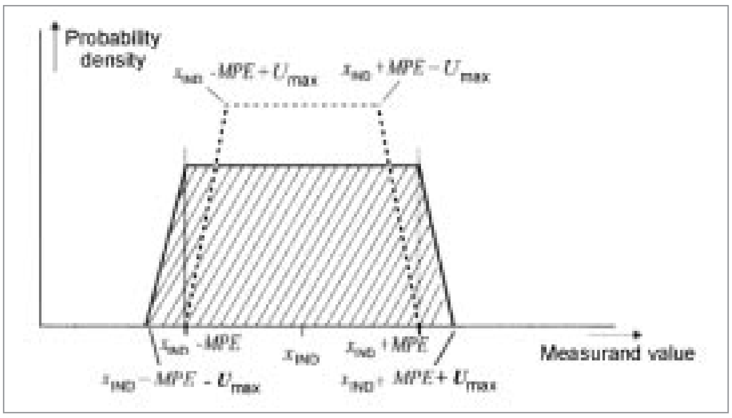

Fig. 7 Suggested probability distributions for evaluating the standard uncertainty contributions of verified measuring instruments.

Plot a: for the classical verification approach;
Plot b: for the modern approach.

MPE value of the maximum permissible errors;
x_{IND} indicated value;
$U_{\max}$ see equation (6).

In the case of a verified or conformity tested instrument, only the positive statement of conformity, the legally prescribed error limits and the decision criteria are known. With regard to the quantity X_{Delta} (see equation (8)), the following is known:

- Classical verification approach to deciding on conformity:

  $|\Delta x| \leq MPE$ and $U_{max} = MPE / 3$

- Modern approach to deciding on conformity:

  $|\Delta x| \leq MPE - U_{95}$ and $U_{max} = MPE / 3$

In both cases, the quantity X_{Delta} (see equation (8)) may be understood as an unknown measurement error, δX_{Delta}, inside the above given limits.

For verified instruments, equation (10) becomes:

$$u(y) = \sqrt{u^2(\delta x_M) + u^2(\delta x_{Delta})} \qquad (10a)$$

The contribution $u(\delta x_M)$ must be estimated in the same way as for calibrated instruments.

$u(\delta x_{Delta})$ can be estimated on the basis of the following knowledge:

- Indications in the ranges of values

  $[y - MPE; y + MPE]$, for the classical approach,

or

  $[y - MPE + U_{max}; y + MPE - U_{max}]$, for the modern approach, can be assumed to be equally probable.

- The probability of indications beyond these intervals declines in proportion to the increase in distance from these limits. Indications outside the intervals $[y - MPE - U_{max}; y + MPE + U_{max}]$, for the classical approach, and $[y - MPE; y + MPE]$, for the new approach, are unlikely.

This knowledge corresponds more or less to a trapezoidal probability distribution as shown in Fig. 7.

Therefore, the uncertainty contribution of newly verified measuring instruments may be estimated by

$$u(\delta x_{Delta}) = a \cdot \sqrt{(1 + \beta^2) / 6} \qquad [5] \qquad (11)$$

Where:

for the classical approach,
$a = U_{max} + MPE; \beta = 0.7\bar{5}$, and,

for the modern approach,
$a = MPE; \beta = 0.60 \ldots 0.80$.

As a result we obtain $u(\delta x_{Delta}) \approx 0.7 \cdot MPE$ (classical approach) and $\approx 0.5 \cdot MPE$ (modern approach).

It should be emphasized that in comparison with calibration results, simplicity and confidence in conformity statements which are provided to the instrument user must be "bought" by keeping a considerable "error reserve". This "error reserve" corresponds to the ratio of maximum permissible errors to the maximum acceptable expanded uncertainty. It also depends on the methodology used to consider the measurement uncertainty.

References

[1] ISO 9001: Quality management systems – Requirements, International Organization of Standardization (ISO), Geneva 2000.

[2] ISO/IEC 17025: General requirements for the competence of testing and calibration laboratories, 2000.

[3] International Vocabulary of Basic and General Terms in Metrology, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1993.

[4] Geometrical Product Specification (GPS) – Inspection by measurement of workspieces and measuring equipment; Part 1: Decision rules for proving conformance or nonconformance with specifications, ISO 14253 – 1, 1998, International Organization for Standardization (ISO), Geneva 1998.

[5] Guide to the Expression of Uncertainty in Measurement, BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, Geneva 1995 (corrected and reprinted).

[6] K.-D. Sommer and S.E. Chappell and M. Kochsiek, "Calibration and verification: Two procedures having comparable objectives and results", OIML Bulletin, Vol. XLII, pp. 5 – 12, January 2001.

[7] OIML R 111: Weights of classes E_1, E_2, F_1, F_2, M_1, M_2, M_3, OIML, 1994.

[8] The Expression of Uncertainty in Quantitative Testing, EA – 3 / 02, Ed. 1, European Cooperation for Accreditation (EA), April 1997

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