VERIFICATION / WEIGHING

A combinatorial technique for weighbridge verification

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Abstract

A general technique for the calibration of metric instruments developed at the Measurement Standards Laboratory of New Zealand is applied to the verification of vehicle weighbridges. The technique, called the combinatorial technique, is used to determine both the errors in the weighbridge scale over the verification range and the associated measurement uncertainty. Using suitable equipment, the measurements can be carried out in a time comparable to that of current techniques. The technique has the advantage that the total mass of the standard weights used can be between 5 % and 50 % of the capacity of the weighbridge. Although reducing the proportion of standard weights increases the uncertainty in calculated scale errors, the technique has sufficient statistical rigor to allow a determination of the degree of confidence in any compliance/non-compliance decision. Examples of the verification of road weighbridges, up to 40 t, using the technique are given.

Keywords: Mass, Weighbridge, Verification

1 Introduction

Ongoing verification of road and rail weighbridges for market surveillance requires regular maintenance, transportation and use of standard weights of large nominal values, typically between 0.1 t and 1 t. A weighbridge can have a capacity of up to 120 t or more, so that verification requires the use of specialized lifting and transportation equipment. Recent developments [1,2] have focused on designing such equipment to minimize the number of personnel required to carry out verification and to improve the efficiency of the verification. Such equipment consists of a truck/trailer system that can transport the standard weights required as well as a forklift and hydraulic hoist for manipulating the weights.

Often it is not possible, practical or legal to transport standard weights that reach the capacity of the weighbridge, in which case verification is achieved by using substitution material [3] instead of standard weights. In general the truck/trailer unit itself is designed to be of sufficient mass to be used as a substitution weight. For example the Rhineland-Palatinate vehicle [2] is a self-contained verification system consisting of a 12.5 t tractor, 15 t trailer, and 27.5 t of standard weights, allowing verification of weighbridges of up to 55 t. Often vehicles or material present at the weighbridge site at the time of the verification are also used as substitution material.

OIML R 76-1 [3] allows the quantity of standard weights required for use in the substitution technique to be as small as 20 % of the capacity of the weighbridge. The use of the substitution technique can therefore be of considerable advantage to a Verification Authority with limited resources. However, as the quantity of standard weights used is reduced, the cumulative effect of errors due to measurement reproducibility increases. Tight constraints are therefore placed on the allowable limits for repeatability error [3], so that the use of the substitution technique in accordance with OIML R 76-1 is often not possible.

In this paper the authors describe the application of a relatively new technique in which the total mass of standard weights required can be reduced to 5 % of the capacity of the weighbridge, while at the same time providing a rigorous analysis of uncertainties in the verification to allow an assessment of the risk arising from using a smaller total mass of standard weights. This technique, called the combinatorial technique, was originally developed for the calibration of resistance bridges used in thermometry [4], but its application to metric instruments in general soon became apparent [5]. The combinatorial technique has practical advantages in large mass and balance calibration [6], and these advantages, with particular regard to weighbridge verification, are discussed here.

In Section 2 of this paper the authors describe the principle of the combinatorial technique. In Section 3 they illustrate the use of the technique with three examples and compare the results of measurements on weighbridges using the combinatorial technique and the substitution technique. In Sections 4 and 5 the practical and theoretical aspects of the technique are considered, and conclusions are given in Section 6.

In this paper the term "reproducibility" rather than "repeatability" is used to describe apparent random variations in measurements. Repeatability, in relation to weighbridges, is defined in OIML R 76-1 as the "ability of an instrument to provide results that agree one with the other when the same load is deposited several times and in a practically identical way on the load receptor under reasonably constant test conditions". This definition is based on that given in the *Guide to the expression of uncertainty of measurement* [7]. However, in the combinatorial technique, the loads used are loaded in different positions and sequences, so that measurement variability is influenced by instrument repeatability as well as eccentric loading and discrimination. These factors combined influence what is referred to here as reproducibility. Also, in this paper the authors use "mass" to mean "conventional value of mass" [8].

2 Description of the technique

The combinatorial technique involves placing m distinct loads in all possible combinations onto the weighbridge. Only one of these loads need consist entirely of standard weights, and the remaining loads are made up with suitable material and vehicles that are available on-site. This gives a total of 2^m possible loading combinations, including the weighbridge zero where no load is used. The masses of the loads are chosen so that the range of combinations covers the operating range of the weighbridge. If *Max* is the maximum capacity of the weighbridge, then a binary sequence of loads having masses of approximately 0.5 *Max*, 0.25 *Max*, 0.125 *Max*, ... gives a uniform coverage of the scale range. In practice 5 loads are usually sufficient, ranging in mass from approximately 0.05 *Max* to 0.5 *Max*. Although the binary sequence is ideal, any sequence of loads that gives a suitable distribution of measurements over the required range is sufficient to give a rigorous assessment of errors over the range of the weighbridge scale.

The basis of the combinatorial technique is that a comparison of scale indications for different combinations of loads can give information on the non-linearity of the scale without the need for standard weights. As an illustration, consider the following measurements carried out on a weighbridge with scale interval $d = 20$ kg. A load of approximate mass 20 t gave a reading of

$$r_1 = 20358 \text{ kg} \tag{1}$$

and a load of approximate mass 10 t gave a reading of

$$r_2 = 10082 \text{ kg} \tag{2}$$

A third measurement using these two loads in combination gave a reading of

$$r_{1+2} = 30426 \text{ kg} \tag{3}$$

so that

$$r_{1+2} - (r_1 + r_2) = -14 \text{ kg} \tag{4}$$

Note that each reading has been corrected using the method described in [3] in which weights of mass 0.1 d are applied to determine the value at which the indication changes. If the scale response was linear one would expect (4) to equal zero. The observation that this is not the case demonstrates these three measurements provide information about the non-linearity of the weighbridge scale. Analysis of readings for all 16 possible combinations of 4 loads, nominally 20 t, 10 t, 5 t and 2.5 t, using least-squares estimation, gives information on the non-linearity of the scale over its entire range up to 40 t. If one of the loads consists of standard weights of known mass, scale errors with corresponding uncertainties of measurement can be determined [5,6]. Note that the non-zero result of Equation (4) may also include components due to instrument repeatability, discrimination and eccentricity errors. However, with the large number of different measurements involved in the combinatorial technique, the effect of these components is "randomized" to some extent, and consequently these components are accounted for in an evaluation of measurement reproducibility from the residuals of the least-squares estimation.

In the combinatorial technique, the dependence of the scale error $E(r)$ on the scale indication r is modeled by a polynomial equation, normally a cubic polynomial of the form

$$E(r) = Ar + Br^2 + Cr^3 \tag{5}$$

where A, B and C are constants that are calculated in the least-squares analysis. Figure 1 illustrates the form of results obtained with the combinatorial technique. The solid curve is the calculated cubic polynomial $E(r)$, and the dashed curves (with light shading between) represent the confidence interval associated with the expanded uncertainty $U(r)$ [7], normally calculated for a 95 % level of confidence. The bold solid lines are specified values of maximum permissible error (MPE) for the device. In the unshaded region of Figure 1, the envelope $E(r) \pm U(r)$ of probable error values lies entirely within the MPE, so that compliance to the MPE can be asserted with a high degree of confidence. Conversely, in the heavily shaded region on the right hand side of Figure 1, the envelope of probable error values lies entirely outside the MPE, so that non-compliance can be asserted with a high degree of confidence. In the shaded region in between, a decision on compliance or non-compliance can only be made with a lesser degree of confidence. However, it is not within the scope of this paper to discuss the assessment of the risk associated with such decisions. The important point to note is that the combinatorial technique gives sufficient statistical information to allow an evaluation of the risk associated with any compliance/non-compliance decision, particularly in situations where the total mass of standard weights available is much less than the capacity of the weighbridge.

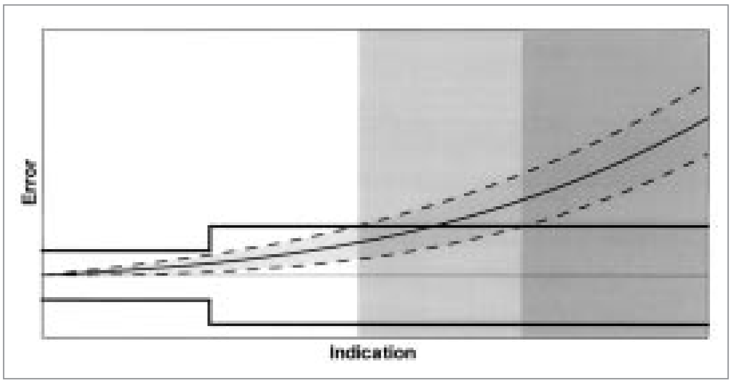

Figure 1 Schematic illustration of the form of results obtained using the combinatorial technique, showing calculated error (solid curve) with associated expanded uncertainties (dashed curves, generally for a 95 % level of confidence). The bold solid lines are the relevant values of MPE.

3 Examples

The three examples presented here describe measurements done during verifications of three different truck weighbridges, each having a scale interval d = 20 kg. In each example, MPE values shown are for a Class III device on subsequent verification, as described in [3]. All weighbridges were verified up to 40 t, which is currently the legal limit for road usage in New Zealand. Also, for each example, measurements using the substitution technique were carried out on the same day, in order to demonstrate the validity of the combinatorial technique. For both techniques, all readings were corrected using the method described in [3], in which weights, of mass 0.1 d, are applied to determine the value at which the indication changes.

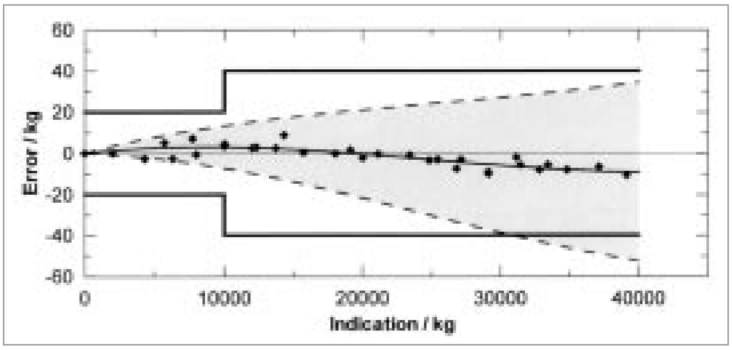

Figure 3 Results of measurements using the combinatorial technique as described in Example 1, using 2 t of standard weights. The data points indicate the variation of the data about the calculated error (solid curve). The dashed curves are the expanded uncertainty in the calculated error, for a 95 % level of confidence. The solid bold lines are the relevant values of MPE.

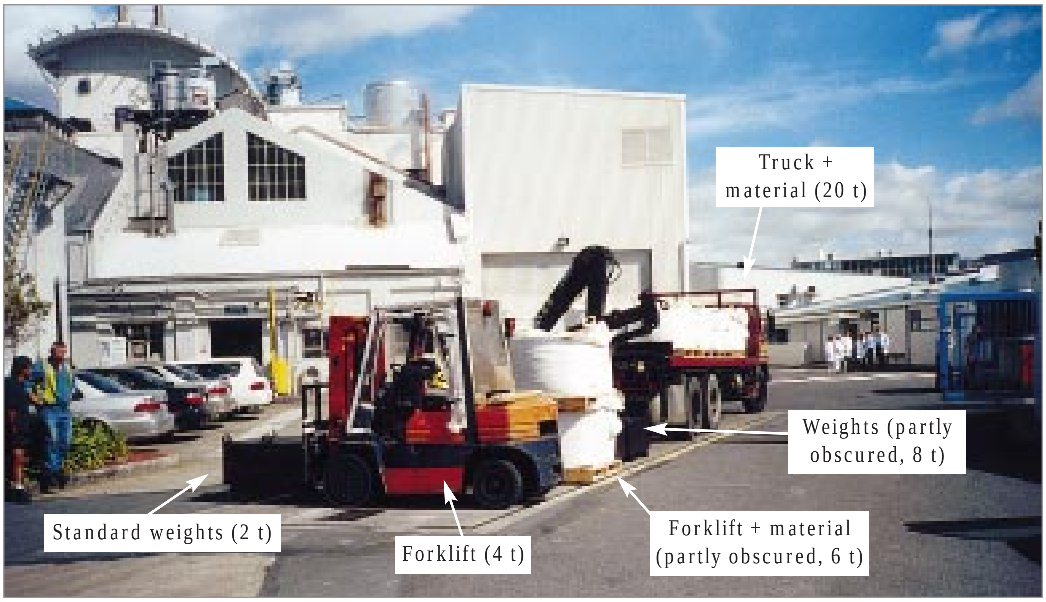

Figure 2 Loads used in the measurements in Example 1.

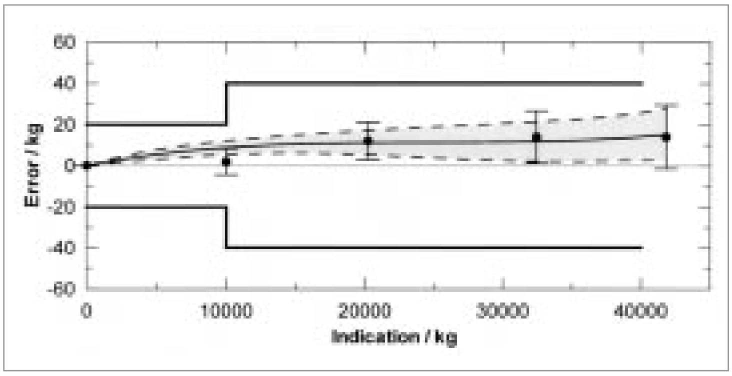

Figure 4 Re-calculated error (solid curve) for the measurements in Example 1, based on 8 t of standard weights, and associated uncertainty (dashed curves). The data points with uncertainty bars are the errors calculated using the substitution technique. All uncertainties are expanded uncertainties for a 95 % level of confidence. The solid bold lines are the relevant values of MPE.

3.1 Example 1

For this weighbridge, of capacity 60 t, measurements were carried out using the combinatorial technique up to 40 t with $m = 5$ loads, made up from vehicles and material available on site, as well as standard weights. Apart from the standard weights, the masses of the loads only need to be known approximately in order to ensure that the combinations are suitable. The only other requirement of the loads is that they be stable over the period of measurements. The loads used in this example were: truck + material (approximate mass 20 t), spare weights (8 t), forklift + material (6 t), 2nd forklift (4 t) and standard weights (2 t). These are shown in Figure 2. Figure 3 shows the results using the combinatorial technique, based on the known mass of the 2 t load of standard weights only. In Figure 3 the solid curve is the least-squares estimate (the calculated error $E(r)$). The data points indicate the variations in the data about $E(r)$ (the "residuals" of the least squares estimation), and these variations are used to determine the reproducibility of the measurements [5,6]. For these measurements, the reproducibility, calculated as a standard uncertainty [7], is $u_R = 3.1$ kg. The reproducibility and the uncertainty in the combination of standard weights are incorporated into the least-squares analysis to calculate uncertainties in the calculated errors $E(r)$ [6]. All other possible uncertainty contributions are negligible, and in the three examples in this paper the uncertainty is dominated by the reproducibility component. This is not entirely obvious from Figure 1, particularly at higher values of scale indication where the variation in the data about the least-squares estimate is small compared to the expanded uncertainty (dashed lines in Figure 3). An inherent characteristic of the combinatorial technique is that the uncertainty in the calculated scale error at a given scale indication is proportional to the product of the reproducibility and the ratio of the indication to the mass of standard weights (see Equation (6) later).

Clearly, from Figure 3, one can assert to a high level of confidence that the errors in the weighbridge indication are within the specified values of MPE. This is a remarkable result, given that the mass of the standard weights used corresponds to 5 % of the capacity of the weighbridge. To demonstrate the dependence of results

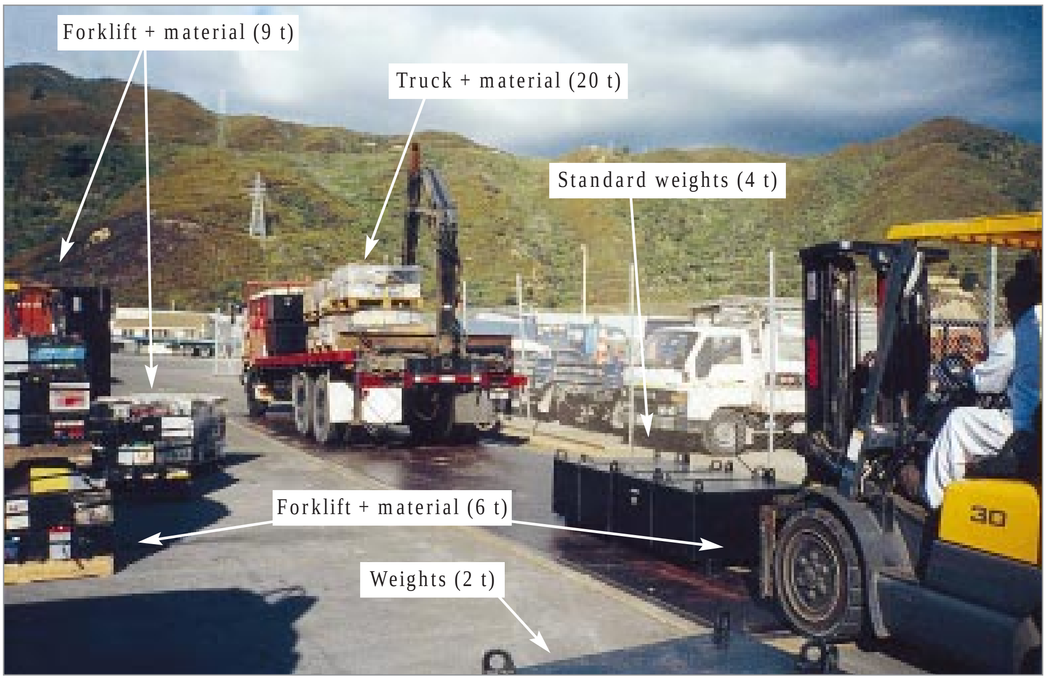

Figure 5 Loads used in the measurements in Example 2.

on the total mass of standard weights, the data was re-analyzed based on the 8 t combination of standard weights, and the results are shown in Figure 4 (data points have been omitted for clarity). Comparing Figures 3 and 4, the uncertainty has been reduced by a factor of four through using 8 t rather that 2 t of standard weights, and the two results show excellent agreement within the calculated uncertainties. Figure 4 also compares the results for the combinatorial technique with those for measurements carried out using the substitution technique. For the substitution technique, 10 t of standard weights were used in 4 substitutions, and the uncertainty limits shown are calculated from the reproducibility determined by the combinatorial technique (see reference [6]). There is excellent agreement between the two techniques. However, it is important to realize that without the estimate of the reproducibility obtained from the combinatorial technique, a proper comparison of the two techniques would not be possible.

3.2 Example 2

For this weighbridge, of capacity 60 t, measurements were carried out using the combinatorial technique up to 40 t with m = 5 loads, made up from vehicles and material available on site, as well as standard weights. The loads were: truck + material (approximate mass 20 t), forklift + material (9 t), 2nd forklift + material (6 t), standard weights (4 t) and spare weights (2 t). These are shown in Figure 5. This verification was based on the 4 t load of standard weights, and although measurements were hindered by windy conditions at the time, the reproducibility was good (u_R = 4.2 kg). Results are shown in Figure 6, along with the results from the substitution technique using 10 t of standard weights. Based on the results of the combinatorial technique, one can assert with a high degree of confidence that the weighbridge complies with the specified MPE. This is confirmed by the excellent agreement with the results of measurements using the substitution technique.

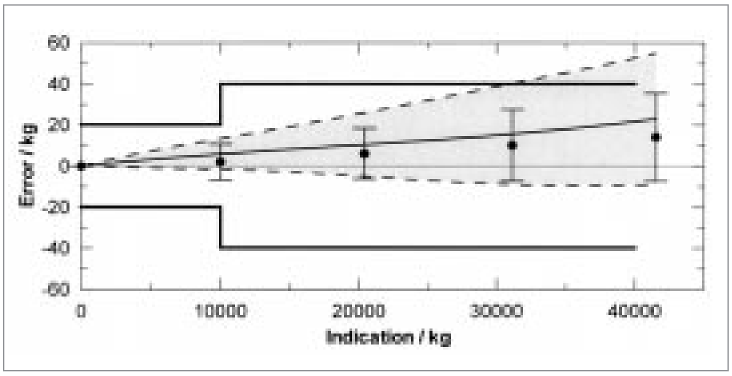

Figure 6 Results of measurements using the combinatorial technique as described in Example 2, using 4 t of standard weights, showing the calculated error (solid curve) and associated uncertainty (dashed curves). The solid bold lines are the relevant values of MPE, and the data points with uncertainty bars are results of measurements using the substitution technique. All uncertainties are expanded uncertainties for a 95 % level of confidence.

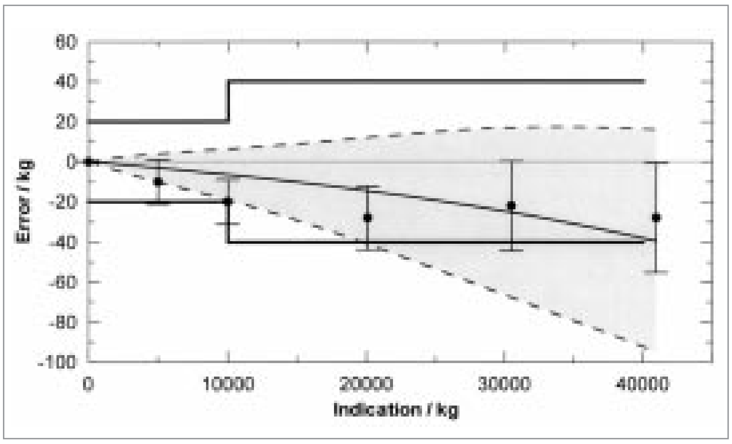

Figure 7 Results of measurements using the combinatorial technique as described in Example 3, using 4 t of standard weights, showing the calculated error (solid curve) and associated uncertainty (dashed curves). The solid bold lines are the relevant values of MPE, and the data points with uncertainty bars are results of measurements using the substitution technique. All uncertainties are expanded uncertainties for a 95 % level of confidence.

3.3 Example 3

For this weighbridge, of capacity 60 t, measurements were carried out using the combinatorial technique up to 40 t with m = 4 loads, made up from vehicles and material available on site, as well as standard weights. The loads were: truck + material (approximate mass 20 t), 2nd truck (10 t), forklift + material (6 t), and standard weights (4 t). Results are shown in Figure 7, along with the results from the substitution technique. The calculated reproducibility was u_R = 4.9 kg. In this example, for the results obtained using the combinatorial technique, the uncertainty is much larger compared with the earlier examples, exceeding the MPE at larger load. This is largely due to the fewer number of combinations used and also the poorer reproducibility. Based on these results, one can only assert that the weighbridge complies with the specified MPE up to around 20 t. As in the previous examples, there is good agreement with the results obtained using the substitution technique.

4 Practical aspects

4.1 Calculations

The least-squares analysis required in the combinatorial technique uses matrix algebra for calculation of scale errors and corresponding uncertainties [6]. These calculations can easily be implemented in computer spreadsheet software such as Microsoft Excel. The calculations for the examples presented here were done using a spreadsheet that is set up so that, once all data are entered into the appropriate cells, the scale errors are automatically calculated. This implementation has the advantage that the operator does not need to fully understand the details of the calculation.

An important advantage of the combinatorial technique is that the reproducibility is assessed from a large number of different combinations of loads. This gives a reliable estimate of the weighbridge reproducibility, as it includes variations that occur due to such effects as repeatability, discrimination and eccentric loading.

4.2 Loading sequences

Table 1 shows the sequence of measurements in Example 2, in the order in which they were carried out. This order was designed to reduce the amount of time and manipulation of loads required. For convenience, the sequence was divided into sub-sequences involving 3 or 4 loadings. The strategy was to keep the larger loads in place while going through the combinations of smaller loads. For example, for the first 4 sub-sequences the truck was left in position on the weighbridge while the other loads were moved on and off and measurements made.

4.3 Resources required

A critical aspect in assessing the practicality of the combinatorial technique is the resources required, in particular time, equipment and number of personnel. In the case where the total mass of standard weights available is less than 10 % of the capacity of the weighbridge, the combinatorial technique requires a similar number and similar types of loadings as the substitution technique [6]. In general, the efficiency of the combinatorial technique is greatly increased if "rolling" loads are used. For example, the use of two forklifts (with skilled drivers) and a truck in examples 1 and 2 allowed efficient manipulation and interchanging of loads. The ideal requirements for a weighbridge verification using the combinatorial technique are given in Table 2. With such equipment available, measurements on a weighbridge using the substitution technique followed by the combinatorial technique were completed within half a day, including the time taken to organize suitable vehicles and material for the loads required. With suitable equipment, measurements using the combinatorial technique can be carried out in a similar time to other current techniques.

5 Theoretical aspects

Although the least-squares analysis will always produce uncertainties for a given set of measurements, it is useful to know in advance what uncertainties can be achieved in a given situation. This can be achieved using the following equation, which gives an approximation for the standard uncertainty $u(r)$ in the calculated error $E(r)$ at a given indication r,

$$u(r) \approx \frac{r}{M}\sqrt{3.6\,\frac{u_R^2}{2^m} + u_M^2} \tag{6}$$

where u_M is the standard uncertainty in the mass M of standard weights. This equation was derived empirically by numerical analysis, and is a slightly better approximation than that given in [6]; it gives values of uncertainty that are within 10 % of those calculated by least-squares analysis, provided that the load of standard weights is either of the two smallest loads used.

Equation (6) can be simplified with the following considerations. It is usually best to use $m = 5$ loads, and the uncertainty u_M in the standard weights is generally small enough to be disregarded. For a properly installed and serviced weighbridge, based on the results presented here, one would expect that $u_R = 0.25\ d$ in the worst case. Equation (6) then becomes, for $m = 5$,

$$u(r) \approx 0.084\ \frac{rd}{M} \tag{7}$$

or as an expanded uncertainty (for $m = 5$),

$$U(r) \approx 0.17\ \frac{rd}{M} \tag{8}$$

A common criterion used in designing measurements for determining compliance or non-compliance is that $U(r)$ should be less than or equal to one-third of the

Table 1 The sequence of loading combinations used in Example 2, and corresponding indications and mass “Delta” of extra weights required to change each indication (all in kg).

Weights used	1	2	3	4	5
Identifiers	20truck	9fork	6fork	4stds	2stds

Loadings	Indication	Delta	Corrected
20truck+9fork+6fork	35000	16	34994
20truck+9fork+6fork+2stds	36980	12	36978
20truck+9fork+6fork+4stds	38980	10	38980
20truck+9fork+6fork+4stds+2stds	40980	12	40978
20truck+9fork	28800	4	28806
20truck+9fork+2stds	30800	2	30808
20truck+9fork+4stds	32820	18	32812
20truck+9fork+4stds+2stds	34820	16	34814
20truck+6fork	25960	8	25962
20truck+6fork+2stds	27960	6	27964
20truck+6fork+4stds	29960	4	29966
20truck+6fork+4stds+2stds	31960	4	31966
20truck	19800	20	19790
20truck+2stds	21800	18	21792
20truck+4stds	23800	18	23792
20truck+4stds+2stds	25800	20	25790
9fork+6fork	15180	8	15182
9fork+6fork+2stds	17180	8	17182
9fork+6fork+4stds	19180	4	19186
9fork+6fork+4stds+2stds	21180	2	21188
9fork	9020	10	9020
9fork+2stds	11020	10	11020
9fork+4stds	13020	10	13020
9fork+4stds+2stds	15020	6	15024
6fork	6180	16	6174
6fork+2stds	8180	18	8172
6fork+4stds	10180	14	10176
6fork+4stds+2stds	12180	14	12176
2stds	2000	10	2000
4stds	4000	8	4002
4stds+2stds	6000	6	6004

Table 2 Ideal requirements for the verification of a weighbridge, up to 40 t, using the combinatorial technique (see Examples 1 and 2).

Equipment	10 t truck 2 forklifts 15–20 t of material to make up loads 2–8 t standard weights
Personnel	2 forklift/truck drivers 1 verifying officer

MPE. Considering the case where $r = Max$, for which MPE = 2 d (for subsequent verification), then this criterion would be met for $M > 0.25\ Max$. That is, based on the assumptions given here, this criterion would be satisfied for a total mass of standard weights that is as small as 25 % of the weighbridge capacity.

6 Conclusions

This paper describes the application of the combinatorial technique to the verification of truck weighbridges. The combinatorial technique can be used in any weighbridge verification, and is particularly suited to situations where it is not feasible to have standard weights that cover the full range of the weighbridge scale. This technique enables a rigorous determination both of the errors in the weighbridge scale and also of the associated uncertainties, and can be easily and efficiently implemented with the use of "rolling" loads. Comparisons of the results of the combinatorial technique with those of the substitution technique, made possible through use of the reproducibility data obtained from the combinatorial technique, demonstrate the validity of the combinatorial technique. The combinatorial technique provides sufficient information to allow a quantitative assessment of the risk associated in making a compliance/non-compliance decision, particularly when the total mass of standard weights used is much less that the capacity of the weighbridge.

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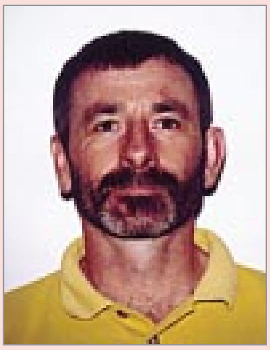

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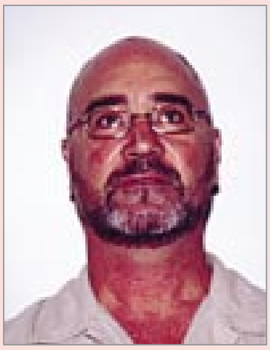

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