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Conventional value of the result of weighing in air

Valeur conventionnelle du résultat des pesées dans l'air

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ORGANISATION INTERNATIONALE
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INTERNATIONAL ORGANIZATION
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Foreword

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Conventional value of the result of weighing in air

0 Introduction

The mass of an object is obtained by weighing in air. Because the weighing instrument indicates a value that is proportional to the gravitational force on the object reduced by the buoyancy of air, the instrument's indication in general has to be corrected for the buoyancy effect. The value of this correction depends on the density of the object and the density of the air.

Conventions have been developed which simplify the mass determination of weights in metrology. A *conventional mass*, m_c, is defined such that, under specified conditions, this air buoyancy correction will be unnecessary. The conditions have been chosen such that mass and conventional mass of a weight do not differ by more than specified maximum values (e.g., see Table 1 in OIML R 111-1 "Weights of classes E_1, E_2, F_1, F_2, M_1, M_{1-2}, M_2, M_{2-3} and M_3. Part 1: Metrological and technical requirements").

1 Scope

This Document presents the definition of the quantity "conventional mass" (conventional value of the result of weighing in air) as it is used for the characterization of weights and its relation to the physical quantities mass and density and the evaluation of its uncertainty. The concept of conventional mass aims to simplify the determination of the mass of weights under conditions in ambient air. It implies that conventional mass does not differ from the physical quantity mass by more than certain acceptable values, when determined under specified limits of air density and the material densities of the weights.

2 Terminology

Mass

Physical quantity, which can be ascribed to any material object and which gives a measure of its quantity of matter. The unit of mass is the kilogram.

Density

Mass divided by the volume of a body, given by the formula:

$$\rho = \frac{m}{V}$$

Weight

Material measure of mass, regulated in regard to its physical and metrological characteristics: shape, dimensions, material, surface quality, nominal value, density, magnetic properties and maximum permissible error.

Note 1: The term "weight" is also used as the physical quantity of the gravitational force of a body [1]. From the context it is usually clear in which sense the term is used. If the sense is not clear, one may use the words "weight force" or "weight piece", depending on its meaning.

3 List of symbols

k	coverage factor
m	mass of a rigid body (weight)
m_c	conventional mass (conventional value of the result of weighing in air)
m_0	mass, nominal value of weight
m_r, m_t	mass of reference weight and test weight
m_{cr}, m_{ct}	conventional mass of reference weight and test weight
U	expanded uncertainty (k = 2)
u_b	uncertainty of the buoyancy correction
u_{ba}	uncertainty of the balance
$u_c(m_{ct})$	uncertainty of the conventional mass
$u(\rho_a)$	uncertainty of the air density
$u(\rho_r)$	uncertainty of the density of the reference weight
$u(\rho_t)$	uncertainty of the density of the test weight
u_w	uncertainty of the weighing process
$u(m_{cr})$	uncertainty of the reference standard
V	volume
δm	maximum permissible error for weights
Δm_w	mass equivalent of the indication difference of the weighing instrument
ρ	density
ρ_0	density of air as a reference value equal to 1.2 kg m^{-3}
ρ_a	density of moist air
ρ_{a1}	air density during the (previous) calibration of the reference weight by use of a higher order reference weight
ρ_c	reference density of a weight, i.e. 8 000 kg m^{-3}
ρ_r, ρ_t	density of reference weight and test weight
t_{ref}	20 °C (reference temperature)

4 Definition of conventional mass

The conventional mass value of a body is equal to the mass m_c of a standard that balances this body under conventionally chosen conditions. The unit of the quantity “conventional mass” is the kilogram (see Note 2).

The conventionally chosen conditions are:

t_{ref} = 20 °C ρ_0 = 1.2 kg m^{-3} ρ_c = 8 000 kg m^{-3}

Note 2: The conventional mass has the same unit as mass, because its values are defined through the multiplication of a mass by a dimensionless quantity.

5 Relationship between the mass and conventional mass of a weight

If the mass of a weight is known (e.g. from a calibration certificate) its conventional mass can be calculated according to this definition from the following formula:

$$m_c = m\frac{1 - \rho_0 / \rho}{1 - \rho_0 / \rho_c} \qquad (1)$$

This definition shows that the conventional mass of a weight made of stainless steel which has a density ρ close to ρ = 8000 kg m^{-3} deviates from its mass by only a small amount. For other materials, the relative deviation of m_c from m ranges from about -3×10^{-4} for aluminium to $+10^{-4}$ for platinum. Nevertheless, the difference between the conventional mass value of a weight and its physical mass can be significant even for weights made of stainless steel (see Example 1, below).

6 Density of weights

Equation (1) is derived from the following weighing equation:

$$m_c(1 - \rho_0 / \rho_c) = m(1 - \rho_0 / \rho) \qquad (2)$$

This equation describes a weighing process, where a weight of mass m and density ρ is in equilibrium with a reference weight of conventional mass m_c and density ρ_c at an air density ρ_0. This is an idealized condition, because real air density usually deviates from ρ_0.

If we assume that the two weights were compared at an air density ρ_a that is different from ρ_0, the weights are no longer in equilibrium and the balance indication will differ from zero by an amount corresponding to Δm_w. We have then:

$$\Delta m_w = m\left(1 - \frac{\rho_a}{\rho}\right) - m_c\left(1 - \frac{\rho_a}{\rho_c}\right) \qquad (3)$$

Equation (3) shows that the weight with mass m, which has a conventional mass m_c derived from the reference mass according to equation (2), is no longer in equilibrium with the reference mass, when the air density deviates from ρ_0. This fact is essential for understanding the utility of conventional mass.

The difference Δm_w is a deviation in a weighing process that leads to an error in determining the conventional mass of a weight when a correction for air buoyancy is not applied (see 7). Because the concept of conventional mass disregards the buoyancy correction, limiting conditions have to be defined. Such limiting conditions concern the density of the weight and the density of air.

If we combine equation (2) with equation (3), we obtain:

$$\rho = \rho_c \frac{\dfrac{\Delta m_w}{m_c} - \dfrac{\Delta\rho_a}{\rho_0}}{\dfrac{\Delta m_w}{m_c}\dfrac{\rho_c}{\rho_0} - \dfrac{\Delta\rho_a}{\rho_0}} \qquad (4)$$

with $\Delta\rho_a = \rho_a - \rho_0$

or, as a reasonable approximation:

$$\rho = \frac{\rho_c}{1 - \frac{\Delta m_w}{m_c}\frac{\rho_c}{\Delta\rho_a}} \tag{5}$$

Requirements for the density of weights

The density of a weight with conventional mass m_c should be such that, at a deviation of the air density $\Delta\rho_a = |\rho_a - \rho_0|$, the relative deviation of the conventional mass $|\Delta m_w / m_c|$ should be smaller than a specified value, laid down in OIML R 111-1 or in an applicable International Standard.

***Example 1** (see [2]):*

Requirements: $$\left|\frac{\Delta m_w}{m_c}\right| \le \frac{1}{4}\frac{\delta m}{m_c}$$

$$\left|\Delta\rho_a\right| \le 0.1\rho_0$$

Density limits:

$$\frac{\rho_c}{1 + \frac{10^5}{6}\frac{\delta m}{m_c}} \le \rho \le \frac{\rho_c}{1 - \frac{10^5}{6}\frac{\delta m}{m_c}} \quad \text{if} \quad \frac{\delta m}{m_c} < 6 \times 10^{-5} \tag{6}$$

$$\frac{\rho_c}{1 + \frac{10^5}{6}\frac{\delta m}{m_c}} \le \rho \quad \text{if} \quad \frac{\delta m}{m_c} \ge 6 \times 10^{-5} \tag{7}$$

In the case of a weight of class F_1 and where m_c = 1 kg and δm = 5 mg, we have:

7 385 kg m^{-3} $\le \rho \le$ 8 727 kg m^{-3}

7 Buoyancy correction

Assuming a test weight has been compared with a reference weight using a weighing instrument, the general weighing equation reads:

$$m_t\left(1-\frac{\rho_a}{\rho_t}\right)=m_r\left(1-\frac{\rho_a}{\rho_r}\right)+\Delta m_w \tag{8}$$

If we replace the masses m_r and m_t by expressions containing the conventional masses m_{cr} and m_{ct} by use of equation (1), we obtain the equation for the buoyancy correction of the conventional mass of the test weight as follows:

$$m_{ct}=m_{cr}(1+C)+\Delta m_w \tag{9}$$

with: $$C=\frac{(\rho_r-\rho_t)(\rho_a-\rho_0)}{(\rho_r-\rho_0)(\rho_t-\rho_a)}$$

which can be approximated, to the first order in the ratios of air densities to weight densities, by:

$$C=(\rho_a-\rho_0)\left[\frac{1}{\rho_t}-\frac{1}{\rho_r}\right] \tag{10}$$

The buoyancy correction is $m_{cr}C$, its relative value being C.

Requirements for the application of a buoyancy correction

A buoyancy correction has to be applied if the absolute value $|C|$ exceeds a specified value, which is laid down in OIML R 111-1 or in an applicable International Standard.

Example 2:

According to [2], a buoyancy correction can be neglected if:

$$|C|\leq\frac{1}{3}\frac{U}{m_0} \tag{11}$$

In other words, a buoyancy correction is unnecessary if $m_0|C|$ is smaller than one third of the expanded uncertainty of the result of measurement, U.

Now consider an example where ρ_r = 7 820 kg m^{-3}, ρ_t = 8 100 kg m^{-3}, and ρ_a = 1.1 kg m^{-3}. It follows from (10) that $|C| = 4.4 \times 10^{-7}$. In the case of a 1 kg weight, a buoyancy correction has to be applied to comply with the requirements of class E_2. This is because [2] specifies that $U \leq 0.5$ mg and this requirement is not consistent with the inequality shown in (11).

8 Uncertainty

The uncertainty of measurement has to be evaluated according to international guidelines, standards or Recommendations. According to [2, 3] the expanded uncertainty U (k = 2) of the conventional mass m_{ct} of a test weight as follows:

$$U = k\, u_c(m_{ct}) \qquad \text{with } k = 2 \tag{12}$$

$u_c(m_{ct})$ is calculated as follows.

With buoyancy correction (equations *(C.6.3-1)* and *(C.6.5-1)* in OIML R 111-1 [2]):

$$u_c(m_{ct}) = \sqrt{u_w^2 + u^2(m_{cr}) + u_b^2 + u_{ba}^2} \tag{13}$$

$$u_b^2 = \left[m_{cr}\frac{(\rho_r - \rho_t)}{\rho_r \rho_t}u(\rho_a)\right]^2 + \left[m_{cr}(\rho_a - \rho_0)\right]^2 \frac{u^2(\rho_t)}{\rho_t^4} + m_{cr}^2(\rho_a - \rho_0)\left[(\rho_a - \rho_0) - 2(\rho_{a1} - \rho_0)\right]\frac{u^2(\rho_r)}{\rho_r^4} \tag{14}$$

Without buoyancy correction:

If the correction is not applied, a corresponding contribution for buoyancy has to be added to the combined uncertainty - in addition to u_b:

$$u_c(m_{ct}) = \sqrt{u_w^2 + u^2(m_{cr}) + u_b^2 + (m_{cr}C)^2 + u_{ba}^2} \tag{15}$$

Note 3: If the air density at the previous calibration of the reference weight ρ_{a1} is not known, it may be estimated from the altitude of the laboratory which calibrated the reference weight (see equation (14)).

Note 4: In order to use equation (14), $u(\rho_r)$ must be known and its value must have been taken into account in the calculation of $u(m_{cr})$.

Note 5: Even if a buoyancy correction is unnecessary, the uncertainty of buoyancy may be non-negligible (see equation (15) and Example 3).

Note 6: Explanatory note to clarify the mathematical background of equation (15).

Variance of a value different from the mean

Expectation of the difference between a statistically distributed quantity X and a given value $\mu + a$ (μ is the expectation of X):

$E\left[(X - (\mu + a))^2\right] = E\left[((X - \mu) + (\mu - (\mu + a)))^2\right]$

$= E\left[(X - \mu)^2\right] - 2aE\left[(X - \mu)\right] + a^2$

$= E\left[(X - \mu)^2\right] + a^2$

because: $E\left[(X - \mu)\right] = 0$

If μ, the expectation of X, is taken as the result, the variance of this result is the expectation $E\left[(X - \mu)^2\right]$.

In a real measurement, this corresponds to the variance (squared standard uncertainty) u^2 if the mean, for example m_{ct}, is taken as the result.

If a value $\mu + a$, different from the expectation μ, is taken as the result, the variance of this result is the expectation $E\left[(X - (\mu + a))^2\right] = E\left[(X - \mu)^2\right] + a^2$.

In a real measurement, this corresponds to the variance $u^2 + (Cm_{cr})^2$ if the value $m_{ct} - Cm_{cr}$ is taken as the result (see equation (9)).

No assumption about the statistical distribution is necessary for this consideration. The only condition is that u^2 is the experimental variance of m_{ct}, when m_{ct} is taken as the mean in a statistical evaluation. u^2 may also be replaced by a combination of experimental variances, statistically or not statistically evaluated.

Example 3:

A class F_1 weight of 1 kg (δm = 5 mg, $U \leq 1.5$ mg) [2] and:

ρ_r = 7 800 kg m^{-3} $U(\rho_r)$ = 140 kg m^{-3}

ρ_t = 8 400 kg m^{-3} $U(\rho_t)$ = 140 kg m^{-3}

ρ_a = 1.16 kg m^{-3} $U(\rho_a)$ = 0.14 kg m^{-3} = (2 × 0.12/√3) kg m^{-3} (see equation *(C.6.3-2)* in [2]).

ρ_{a1} = 1.2 kg m^{-3} $U(\rho_{a1})$ = 0.0012 kg m^{-3}

m_{cr} = 1 kg

We find from equations (10) and (14): $|C| = 3.7 \times 10^{-7}$, u_b = 0.64 mg and $m_{cr}\,|C|$ =0.37 mg.

A buoyancy correction is not necessary according to equation (11) and will not be applied. The uncertainty contribution however in this case is:

$$\sqrt{u_b^2 + (m_{cr}C)^2} = 0.7 \text{ mg}$$

which is non-negligible because it corresponds to an expanded uncertainty of U = 1.4 mg, if all other uncertainty contributions were negligibly small, and because the upper limit of the test weight's expanded uncertainty is 1.5 mg.

9 Weighing of bodies

In general, the mass of a body can be determined either:

a) By comparison with a weight or a mass standard as a reference, using a balance or weighing instrument as a comparator, or

b) By using a weighing instrument as a reference instrument.

If weights or mass standards are used as the reference they have to be calibrated or adjusted, thus guaranteeing traceability to the international prototype of the kilogram. If weighing instruments are used, they have to be calibrated or adjusted using traceable weights.

When a body is used in trade, health, the environment or similar fields, the mass of the body is usually determined in agreement with national or international regulations or standards required for weights or for weighing instruments used as references. In other, non-regulated fields, only general scientific rules have to be observed.

If regulations for weights and weighing instruments have to be observed, it is sufficient to consider the conventional mass of the body to be either:

a) The sum of the conventional mass of the reference weight and the observed weighing difference, when a comparison weighing is made, or

b) The indication of the weighing instrument, when it is used as a reference instrument.

A buoyancy correction is not generally necessary.

In cases where such regulations are not valid, or in cases where the physical quantity mass is required instead of the conventional mass value, a buoyancy correction shall generally be applied. Equation (8) is the general weighing equation in terms of the physical quantity mass of a test weight or body and that of a reference mass. Depending on the required uncertainty, the mass may also be calculated from the conventional mass using equation (1) and by inserting the known or assumed density of the body. It has to be considered that the uncertainty of the mass is generally larger than that of the conventional mass of the same body. The reason is that the buoyancy correction for conventional mass refers only to the difference $\rho_a - \rho_0$ (at the place of calibration, see equations (9) and (14)), whereas that for mass refers to ρ_a (see equation (8)). The uncertainty of the mass can be derived from equation (8). It shall be compared with equations (13) and (14) or with equation (15) for calculating the uncertainty of the mass from the conventional mass and its uncertainty. It is obvious that information on the air density during the calibration of the weight as well as on the material densities of the weight and the reference weight and their uncertainties are required for assessing the uncertainty of the mass. If they are not known, reasonable assumptions have to be made.

10 References

[1] *Le Système international d'unités / The International System of Units,* 7th ed., Bureau International des Poids et Mesures, 1998, 152 p. (see p. 42 for official record in French; p. 118 for English translation). The 7th Edition and the 2000 supplement are available on the internet at: *http://www.bipm.org/en/si/si_brochure/*

[2] OIML Recommendation R 111 (Parts -1 and -2), Weights of classes E_1, E_2, F_1, F_2, M_1, $M_{1\text{-}2}$, M_2, $M_{2\text{-}3}$ and M_3 (2004)

[3] Guide to the expression of uncertainty in measurement (GUM), ISO, 1993, Switzerland