International Organization of Legal Metrology
Organisation Internationale de Métrologie Légale

Applying Monte Carlo Methods during verification of measuring instruments

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Introduction

- Laptops and PCs more powerful
- Monte Carlo Methods
- Uncertainty analysis
- And more …
  - Confidence level of measurement based decisions
- Examples

Objective: Demonstrate that verification with Monte Carlo is very simple

Monte Carlo – Uncertainty

GUM

(Guide to the expression of Uncertainty in Measurement, 1993)

- Linear approximation: u(x) << x
- Corrections for known deviations

Monte Carlo – Uncertainty – 2

Not mentioned in the GUM:

- Conformity assessment
  - Testing versus tolerances
  - Not correcting for known deviations
- Modelling: e.g. regression
- Propagation of distributions
  - Non-linear models and u(x) = O(x)
- More than one measurand

→ Joint Committee for Guides in Metrology

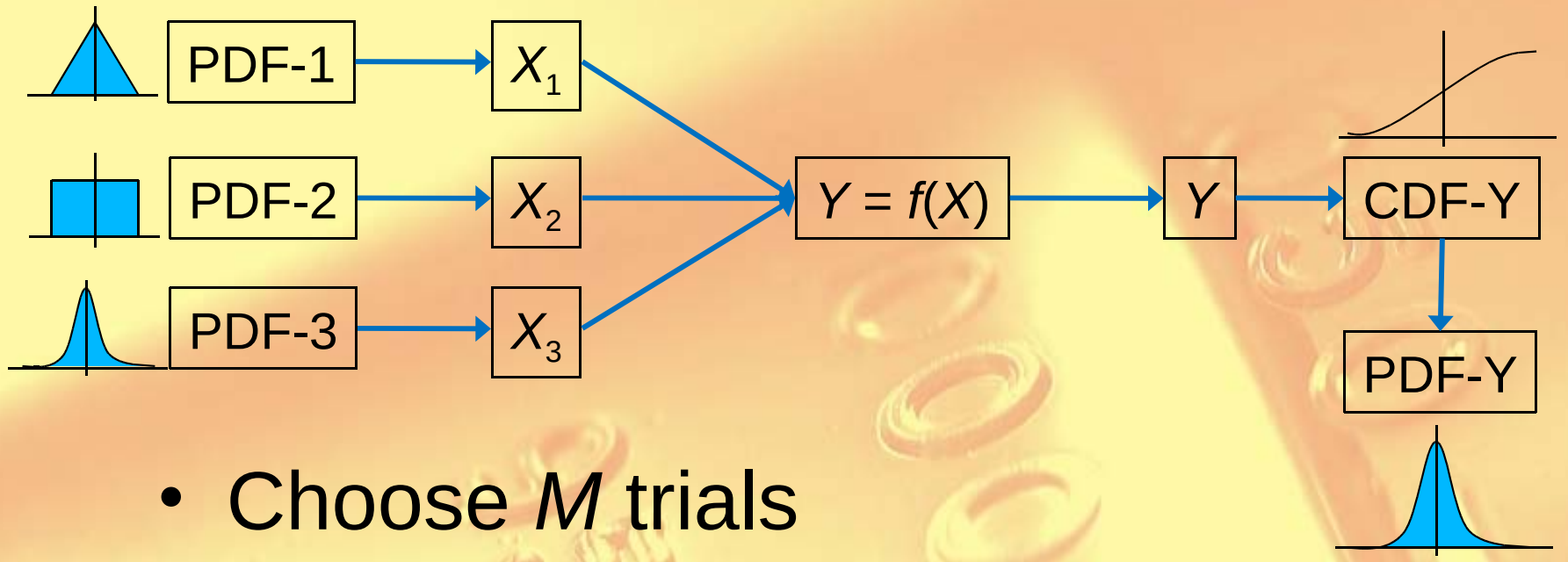
  - Supplementary guides

Monte Carlo – Uncertainty – 3

- JCGM 101 (2008) or OIML G 1-101 (2008)

  Evaluation of measurement data – Supplement 1 to the “Guide to the expression of uncertainty in measurement” – Propagation of distributions using a Monte Carlo method

Monte Carlo – Method – 1



- Choose M trials
- Repeat for $k = 1 .. M$
  - Generate $x_{1,k}$, $x_{2,k}$, … , $x_{N,k}$
  - Calculate $y_k = f (x_{1,k} , x_{2,k} , … x_{N,k})$
- Sort y_k in ascending order → CDF

Monte Carlo – Method – 2

- Estimate of y is average of all y_k
- Associated standard uncertainty is the experimental standard deviation of y_k

CDF

- $P(Y < y_q) = q / M$
- 95% coverage interval $[y_q , y_{q+0,95 \cdot M}]$
- Symmetric coverage $[y_{0,025 \cdot M} , y_{0,975 \cdot M}]$
- Shortest coverage $[y_q , y_{q+0,95 \cdot M}]$ min

Monte Carlo – Method – 3

PDF

- Create histogram
  - Number of bins
  - Number of *y* in each bin
- Scale histogram so that total area is unity

Monte Carlo – Examples

- $Y = X_1 + X_2$
  - Fill out formula '= *x*(1)+*x*(2)
  - Assign Gaussian distribution $\mu_{1,2} = 0$, $\sigma_{1,2} = 1$
- What do we expect for *y* and *u*(*y*)?

- Repeat with rectangular distribution with the same $\mu_{1,2} = 0$, $\sigma_{1,2} = 1$
- Compare 95% coverage intervals for Gaussian and rectangular PDFs

Monte Carlo – Validation

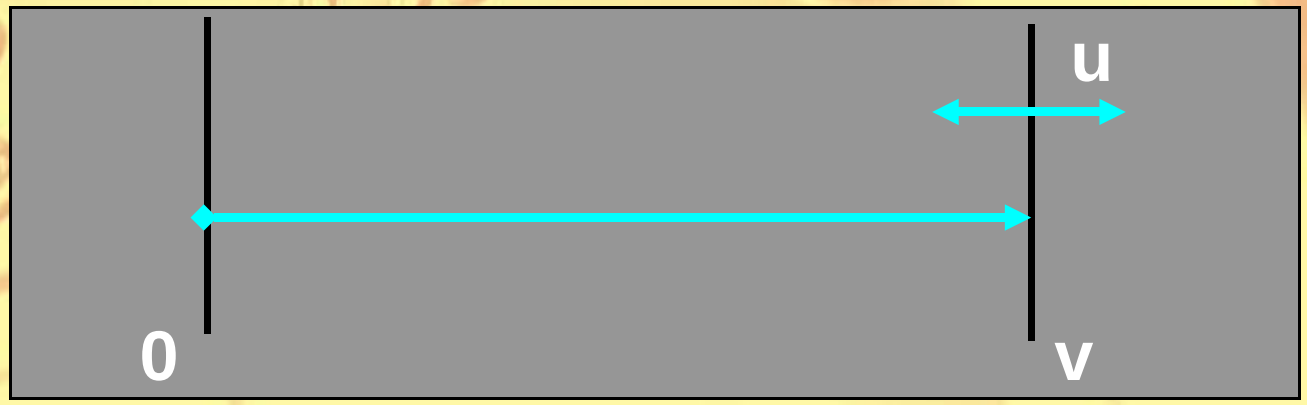
- 10 examples in GUM supplement
- Cases that have been analyzed analytically
  - Flow: master meter method
  - Force: bending an aluminium bar
- Comparison with NPL
  - Entirely different MC implementations
    - Different random number generators
    - Different seeding
  - Comparable results

Monte Carlo – Summary

- Monte Carlo simulations
  - Conceptually difficult
  - Right tool: accurate, easy & fast
  - Non-linear problems
- MCS tool works
  - Software validation completed
- Applications
  - Calibrations
  - Certification of facilities, e.g. EuroLoop

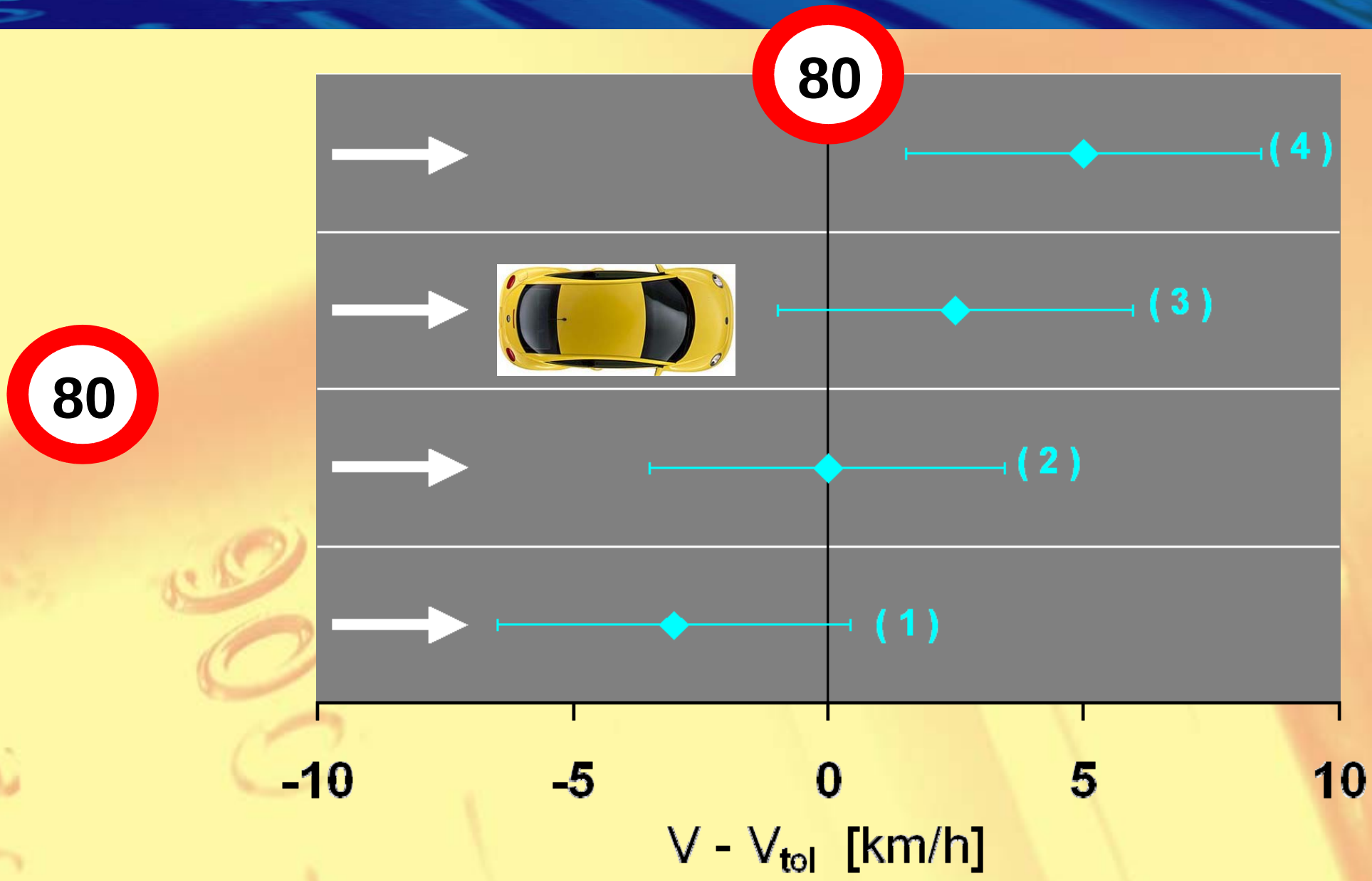
Decisions

- Risk
  probability of taking a wrong decision
- Uncertainty (k=2)
  - range of values that can reasonably be attributed to the measurand
  - Cause for risk on an erroneous decision

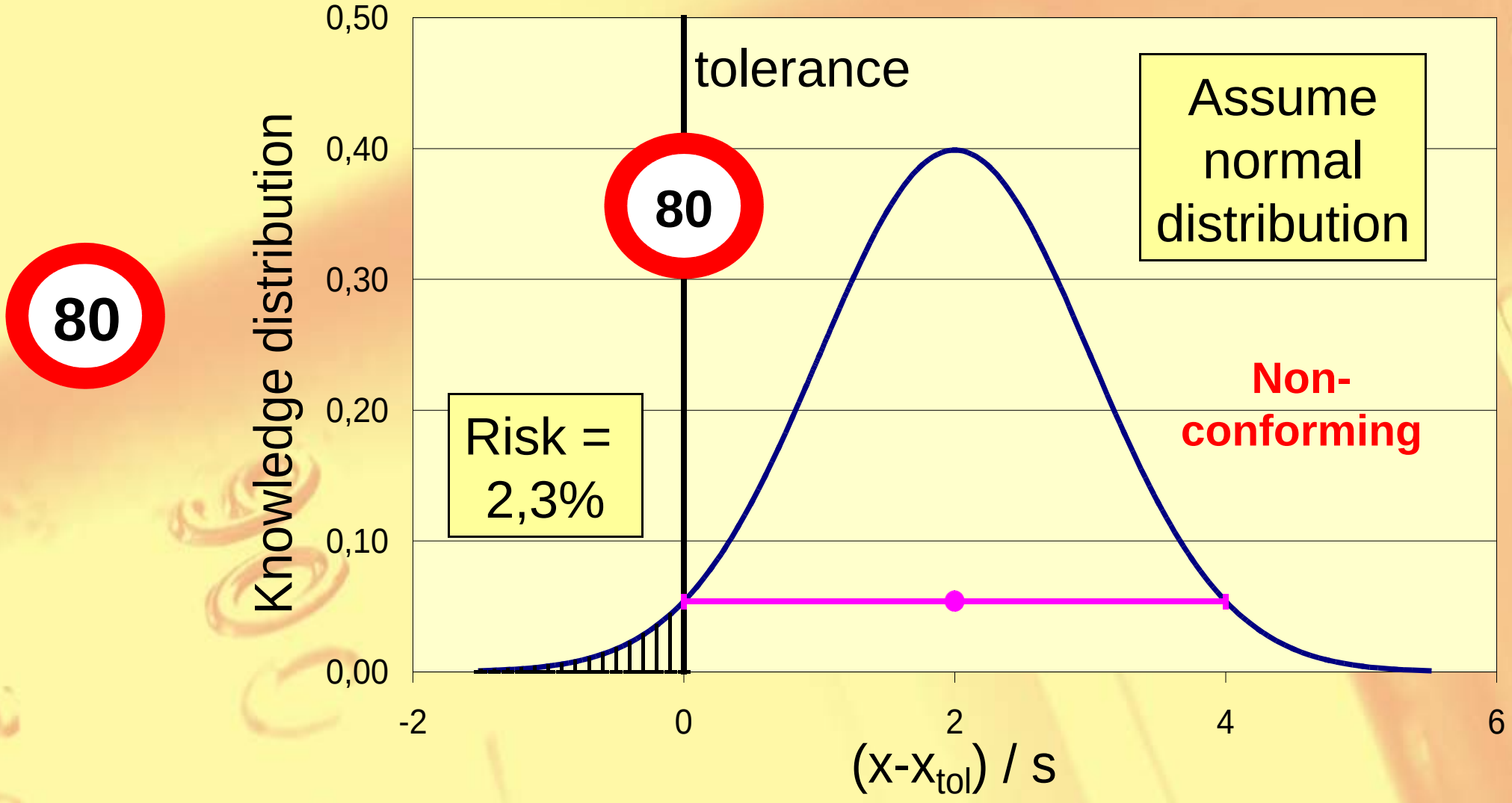


- MID: more parties → identical decision?

Decisions – Driving too fast



Decisions – Driving too fast



Decisions – Driving too fast

- Speed enforcement by radar in NL

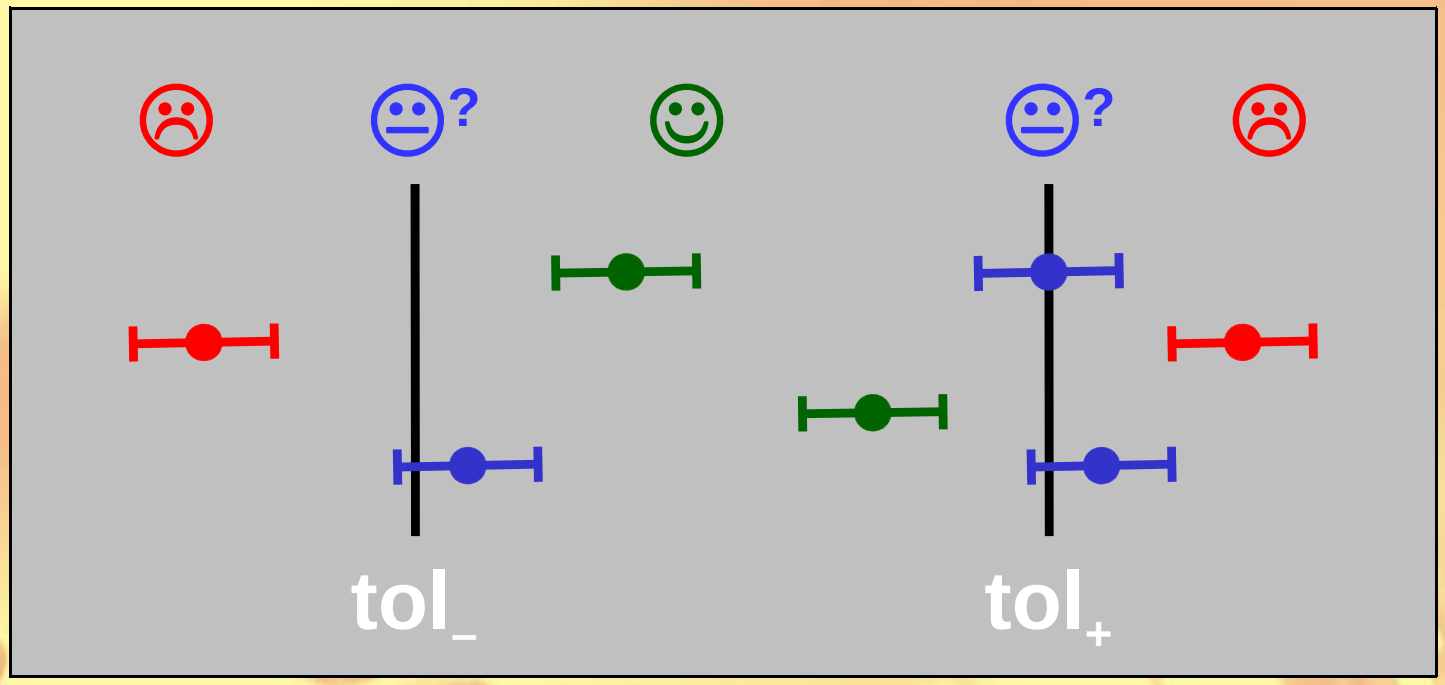
Range	Threshold	MPE	$U_{k=2}$
0 – 100 km/h	7 km/h	3 km/h	3,46 km/h

- MPE subtracted from observation
- Risk

Category	Criterion	Risk
Speed offence	> 4 km/h	< 0,01%
Losing licence	> 50 km/h	< 4,2%

- Acceptable to driver and prosecution?

Conformity assessment



- Test / verification: **conforming** or **not**
- Inspection: **non-conforming** or **not**

Conformity assessment – 2

- Risks accepted in practice

Activity	e ≤	Risk ≤
Type approval	MPE	50%
Type approval	MPE – U	2,3%
Type approval	6/5 MPE – U	21~50%
Init. verification	MPE	50%
Init. verification	MPE – U	2,3%
Init. verification	4/3 MPE – U	25~50%
Re-verification	$MPE_{in\text{-}service}$	50%
Re-verification	$MPE_{in\text{-}service}$ – U	2,3%
Inspections	$MPE_{in\text{-}service}$ + U	2,3%

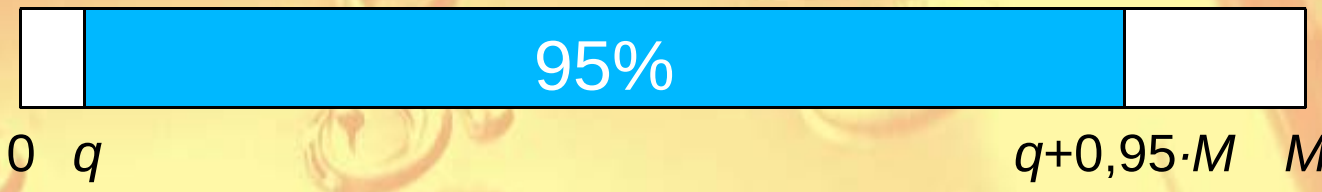
Decisions – Summary

- Perspective
  - Manufacturer or Police
- Confidence level for decisions
  - What risk level is required so that a reliable decision is taken?
  - Huge palette of acceptance criteria to choose from

Verification and Monte Carlo

Remember the CDF

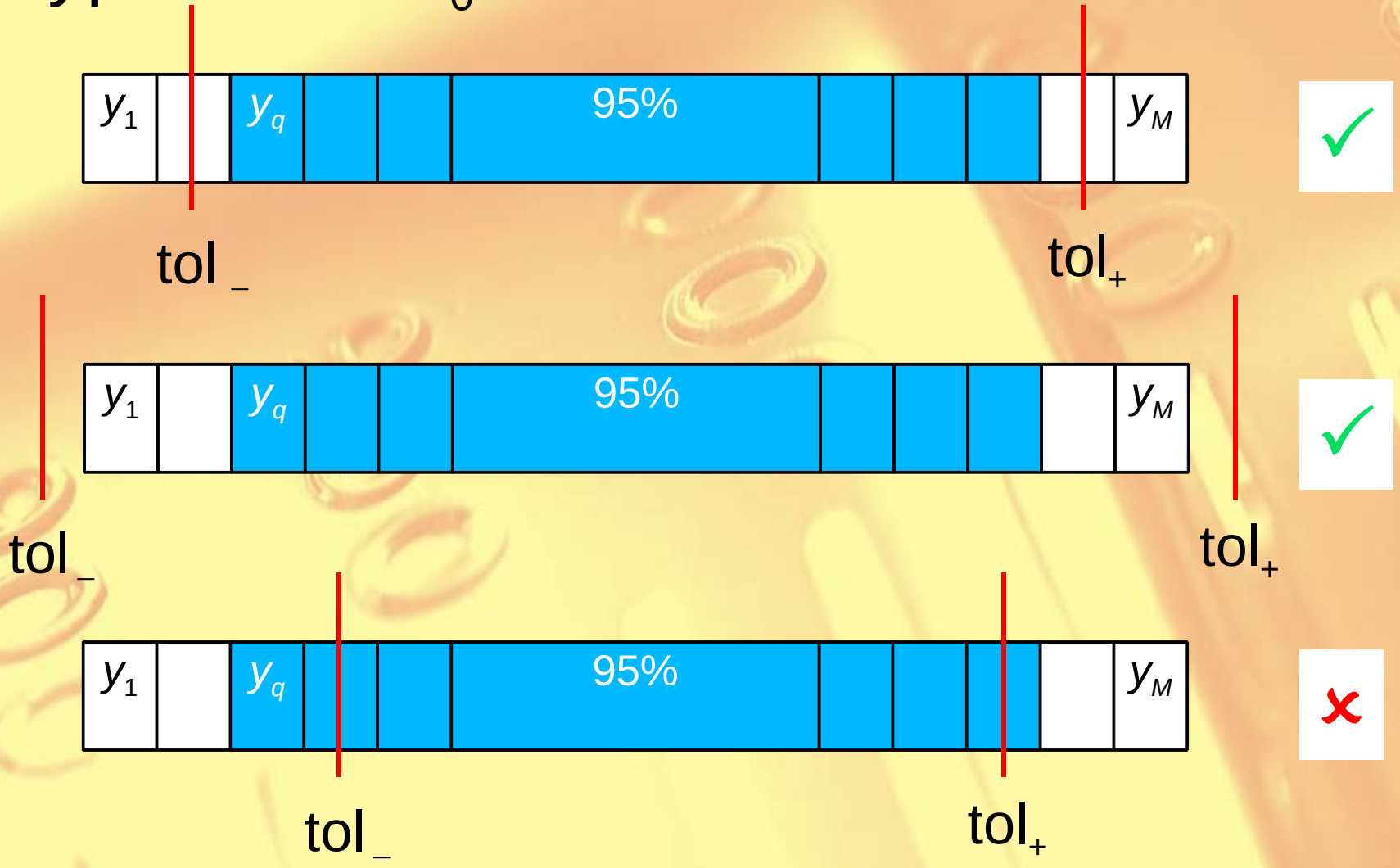
- All y in increasing order
- $P(Y < y_q) = q / M$
- 95% coverage interval $[y_q , y_{q+0,95 \cdot M}]$



- $[y_q , y_k]$ → coverage = $(k-q)/M$ (%)

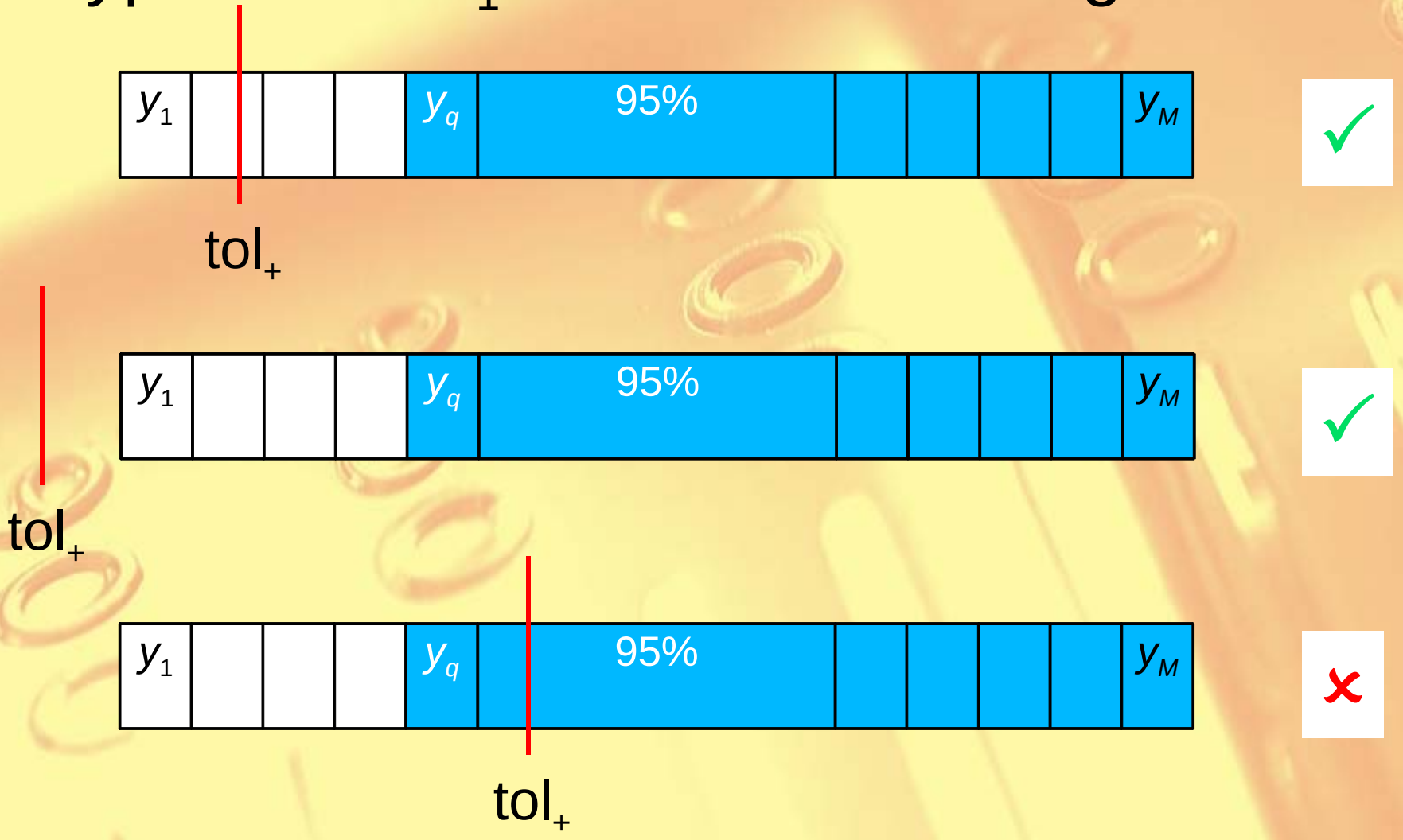
Verification and Monte Carlo

Hypothesis H_0: instrument is OK



Verification and Monte Carlo

Hypothesis H_1: motorist is driving too fast



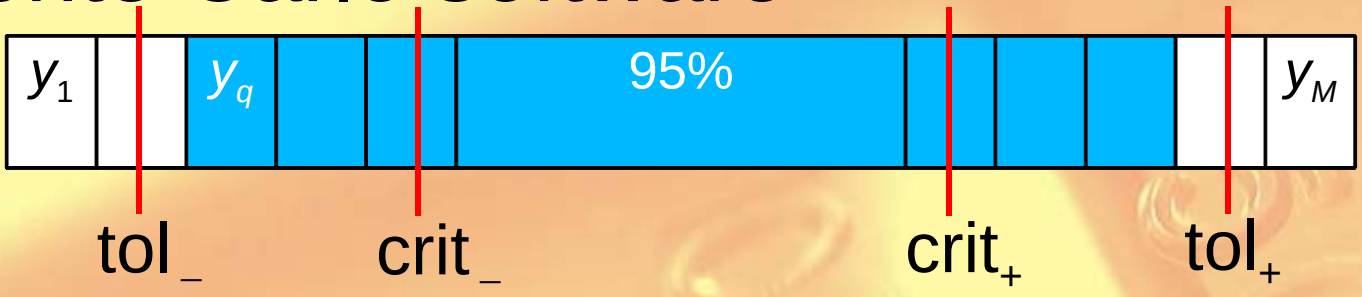
Verification and Monte Carlo

Decision

- Confidence level = 95%, or
- Risk = 5%

Verification and Monte Carlo

Monte Carlo software

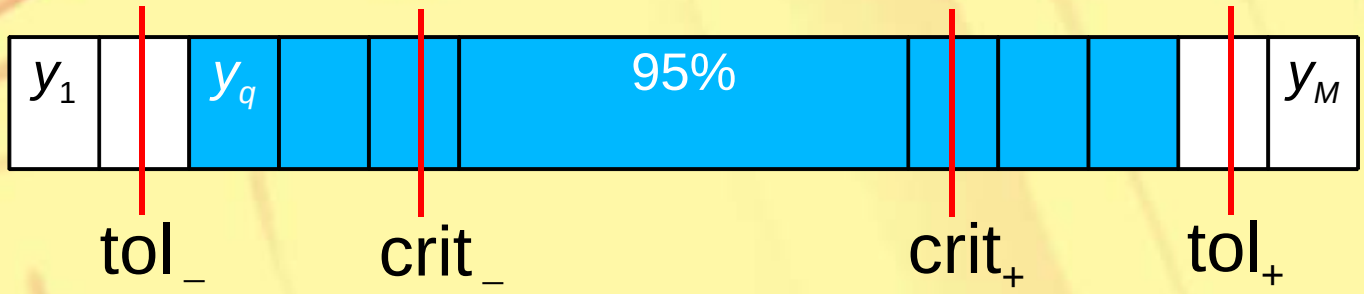


Preset confidence level, e.g. 95%

- H_0:Within criteria
- H_1:Within tolerances
- H_2:Outside tolerances
- H_3:Outside criteria
- Undecided

Conclusions

- Monte Carlo also applicable in legal metrology
- As CDF is already available
- Decision is small addition to the Monte Carlo code
- Easy to operate
- Add to GUM suppl. conformity assess.



Thank you